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$\int x^x (1+\log x)dx$ is equal to

(a) x^x

(b) x^{2x}

(c) $x^x \log x$

(d) $1/2\,(1+\log x)^2$

$\Rightarrow \boxed{x^x = t} \Rightarrow \log x^x = \log t$

$\boxed{x \log x = \log t}$

Derivative $\rightarrow$ $x \cdot \frac{1}{x} + \log x \cdot 1 = \frac{1}{t} \times \frac{dt}{dx}$

$\Rightarrow [1+\log x]\,dx = \frac{dt}{t}$

So: $\int \cancel{t} \cdot \frac{dt}{\cancel{t}}$

$\int 1 \cdot dt = \boxed{t}$

$\Rightarrow x^x$

$$\int \sec^{2/3} x \, \text{cosec}^{4/3} x \, dx =$$

(a) $-3(\tan x)^{1/3} + c$ (b) $-3(\tan x)^{-1/3} + c$

(c) $3(\tan x)^{-1/3} + c$ (d) $(\tan x)^{-1/3} + c$

$\boxed{\tan x \to \sec^2 x}$

$\dfrac{1}{\sec^{2/3}} \times \sec^2 x \times \sec$

$$\to \int \frac{1}{\cos^{2/3} x \cdot \sin^{4/3} x} \cdot dx = \int \frac{\sec^2 x}{\cos^{2/3} x \, \sin^{4/3} x \cdot \sec^2 x} \cdot dx$$

$$\to \int \frac{\sec^2 x \cdot dx}{\sin^{4/3} x \times \cos^{2/3} x \times \cos^{-2} x} = \int \frac{\sec^2 x \, dx}{\sin^{4/3} x \times \frac{1}{\cos^{4/3} x}}$$

$$\Rightarrow \int \frac{\sec^2 x \cdot dx}{\tan^{4/3} x}$$

$$\int \frac{\sec^2 x \cdot dx}{(\tan x)^{4/3}}$$

$\Rightarrow$ let $\boxed{\tan x} = t$

$\sec^2 x \cdot dx = dt$

$$\to \int \frac{1}{t^{4/3}} \cdot dt = \frac{t^{-4/3+1}}{-4/3+1}$$

$$= \frac{-3}{1} \times \frac{1}{t^{1/3}} = \frac{-3}{(\tan x)^{1/3}} + c$$

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$$\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}}\,dx \text{ is equal to}$$

$10x^9$

(a) $10^x - x^{10} + C$ (b) $10^x + x^{10} + C$

(c) $(10^x - x^{10})^{-1} + C$ (d) $\log_e(\log^x + x^{10}) + C$

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$t \Rightarrow 10^x + x^{10}$

$\Rightarrow dt = (10^x \cdot \log_e 10 + 10x^9)\,dx$

$\int \frac{1}{t}\cdot dt = \log t = \log_e(10^x + x^{10}) + C$

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$\int e^x \left(\frac{1-\sin x}{1-\cos x}\right)dx$ is equal to

(a) $-e^x \tan\left(\frac{x}{2}\right)+C$ (b) $-e^x \cot\left(\frac{x}{2}\right)+C$

(c) $-\frac{1}{2}e^x \tan\left(\frac{x}{2}\right)+C$ (d) $\frac{1}{2}e^x \cot\left(\frac{x}{2}\right)+C$

$\int e^x (f(x)+f'(x)) = e^x . f(x) + C$

$\cos 2x = 1 - 2\sin^2 x$

$\sin x = 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}$

let $\boxed{f(x) = -\cot\frac{x}{2}}$

$f'(x) = -(-\operatorname{cosec}^2\frac{x}{2})\times\frac{1}{2}$

$\boxed{f'(x) = \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}}$

$\int e^x\left[\frac{1-\sin x}{2\sin^2 x/2}\right] = \int e^x\left(\frac{1}{2\sin^2 x/2} - \frac{\sin x}{2\sin^2 x/2}\right)$

$\rightarrow \int e^x\left[\frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \frac{2\sin x/2 \cos x/2}{2\sin^2 x/2}\right] \Rightarrow \int e^x\left[\underbrace{\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}}_{f'(x)} - \underbrace{\cot\frac{x}{2}}_{f(x)}\right]$

$\rightarrow \int e^x[f'(x) + f(x)]dx$

$\Rightarrow e^x . f(x) \Rightarrow e^x . (-\cot\frac{x}{2}) + C$

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$$\int \frac{x^9}{\left(4x^2+1\right)^6}\,dx \text{ is equal to}$$

$$\int \frac{x^9}{(x^2)^6\left[4+\frac{1}{x^2}\right]^6}\cdot dx$$

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C$

(c) $\frac{1}{10x}\left(\frac{1}{x}+4\right)^{-5}+C$ (d) $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+C$

$$\Rightarrow \int \frac{1}{x^3\underbrace{\left[4+\frac{1}{x^2}\right]^6}_{t}}\cdot dx \Rightarrow \text{let } 4+\frac{1}{x^2}=t$$

$$0+(-2)x^{-3}\cdot dx = dt \Rightarrow \frac{-2}{x^3}dx = dt$$

$$\rightarrow \frac{dx}{x^3} = \frac{-dt}{2}$$

$$\Rightarrow \int \frac{-dt}{2t^6} = -\frac{1}{2}\int t^{-6}\cdot dt$$

$$= -\frac{1}{2}\,\frac{t^{-5}}{-5} = \frac{1}{10}\times\frac{1}{}$$

$e^{\log x^3}$ $\{ e^{\log} = 1 \}$

$\int e^{3\log x}(x^4+1)^{-1}dx$ is equal to

(a) $\log(x^4+1)+C$ (b) $\frac{1}{4}\log(x^4+1)+C$

(c) $-\log(x^4+1)+C$ (d) None of these

$\int 1\times x^3 (x^4+1)^{-1} \cdot dx$

$\to \int \frac{x^3}{1+x^4} \cdot dx \Rightarrow \int \frac{1}{t} \cdot dt/4$

$1+x^4 = t$ $\boxed{4x^3 \cdot dx = dt} \to \frac{1}{4} \cdot \log|1+x^4|+c$

The value of $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ is

$\rightarrow \int \frac{1}{t}\cdot\frac{dt}{2}$

t

(a) $\frac{1}{2}\log\left|e^{x}+e^{-x}\right|+C$ (b) $2\log\left|e^{2x}+e^{-2x}\right|+C$

(c) $\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$ (d) None of these

$\frac{1}{2}\log|\text{——}|$

Which of the following is/are correct?

I. $\boxed{\int \frac{dx}{x\sqrt{x^2-1}}} = \text{cosec}^{-1}x + C$

$-\text{cosec}^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

II. $\int e^x dx = \log e^x + C$

III. $\int \frac{1}{x} dx = \log|x| + C$

IV. $\int a^x dx = a^x + C$

$a^x/\log_e a$

(a) I and III are correc (b) All are correct

(c) Only III is correct (d) All are incorrect

Match the following derivatives of the functions in column-I with their respective anti-derivatives in column-II.

Column - I	Column - II
A. $\frac{1}{\sqrt{1-x^2}}$	1. $\tan^{-1}x+C$
B. $\frac{-1}{\sqrt{1-x^2}}$	2. $\cot^{-1}x+C$
C. $\frac{1}{1+x^2}$	3. $\sin^{-1}x+C$
D. $\frac{-1}{1+x^2}$	4. $\cos^{-1}x+C$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	3	4	2	1
(c)	3	4	1	2
(d)	4	3	2	1

Match the following integrals in column-I with their corresponding values in column-II.

Column-I		Column-II
A.	$\int \sqrt{ax+b}\,dx$	1. $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$
B.	$\int x\sqrt{x+2}\,dx$	2. $\frac{1}{6}\left(1+2x^2\right)^{3/2} + C$
C.	$\int x\sqrt{1+2x^2}\,dx$	3. $\frac{4}{3}\left(x^2+x+1\right)^{3/2} + C$
D.	$\int (4x+2)\sqrt{x^2+x+1}\,dx$	4. $\frac{2}{3a}(ax+b)^{3/2} + C$

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	3	4	2	1
(c)	1	3	2	4
(d)	3	2	4	1

$\int \frac{1}{\sqrt{a^2+x^2}}\,dx$ ⑥

① $\int \sqrt{a^2-x^2}\,dx = \boxed{\frac{x}{2}\sqrt{a^2-x^2}} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

② $\int \sqrt{x^2-a^2}\,dx = \boxed{\frac{x}{2}\sqrt{x^2-a^2}} - \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + C$

③ $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

1	2	3	4	5	6	7	8	9	10
A	B	D	B	D	B	C	C	C	A