Q. $\int x \cdot \tan^{-1} x \cdot dx$ (I L A T E)

$\Rightarrow \int x \cdot \tan^{-1} x \cdot dx = \tan^{-1} x \cdot \frac{x^2}{2} - \int \left\{ \frac{1}{1+x^2} \times \frac{x^2}{2} \right\} \cdot dx$

$I = \frac{\tan^{-1} x \cdot x^2}{2} - \frac{1}{2} \int \frac{x^2}{1+x^2} \cdot dx$ — 1

$I_1 = \int \frac{x^2}{1+x^2} \cdot dx = \int \frac{1+x^2}{1+x^2} dx \quad \int \frac{1}{1+x^2} dx$

$I_1 = x - \tan^{-1} x$

from eq^n 1 –

$I = \frac{\tan^{-1} x \cdot x^2}{2} - \frac{1}{2}(x - \tan^{-1} x) + C$

H.W → Q ⇒ $\int x \cdot \cos^{-1} x \cdot dx$

$\int x \cdot \sin^{-1} x$.

$\int x \cdot \tan^{-1} x$.

$\sqrt{a^2 - x^2}$

Q. $\int (\sin^{-1}x)^2 \cdot dx =$

$\sin^{-1}x = t \longrightarrow \sin t = x$

$\frac{1}{\sqrt{1-x^2}} dx = dt \Rightarrow \frac{1}{\sqrt{1-\sin^2 t}} \cdot dx = dt$

$dx = dt \times \cos t$

$\Rightarrow \int \underset{①}{(\sin^{-1}x)^2} \cdot \underset{②}{1} \cdot dx$

$\int \underset{①}{t^2} \cdot \underset{②}{\cos t} \cdot dt$

$\therefore \int (\sin^{-1}x)^2 \cdot 1 \cdot dx = (\sin^{-1}x)^2 \cdot x - \int \left\{ \frac{d}{dx}(\sin^{-1}x)^2 \times \int 1 \cdot dx \right\} \cdot dx$

$I = (\sin^{-1}x)^2 \cdot x - \int \left\{ 2 \sin^{-1}x \times \frac{1}{\sqrt{1-x^2}} \times x \right\} \cdot dx$

ILATE

$I_1 = \int \underset{①}{t} \cdot \underset{②}{\sin t} \cdot dt$

So $I_1 = \int \frac{\sin^{-1}x \cdot x}{\sqrt{1-x^2}} \cdot dx \Rightarrow$

$\sin^{-1}x = t \rightarrow \sin t = x$

$\frac{1}{\sqrt{1-x^2}} \cdot dx = dt$

$\hookrightarrow \cos t = \sqrt{1-x^2}$

$$I_1 = \int t \cdot \sin t \cdot dt = -t \cdot \cos t - \int 1 \times (-\cos t) \cdot dt$$

$$I_1 = -t \cdot \cos t + \sin t$$

$$\boxed{I_1 = -\sin^{-1}x \times \sqrt{1-x^2} + x}$$

$$\text{So } I = (\sin^{-1}x)^2 x + 2 \cdot \sin^{-1}x \cdot \sqrt{1-x^2} - 2x + C$$

Q. $\int \underline{(x^2+1)} \cdot \underline{\log x} \cdot dx = \int \underbrace{x^2 \log x \cdot dx}_{I_1} + \int \underbrace{\log x \, 1 \cdot dx}_{I_2}$

$I_1 = \int \underset{①}{\underline{\log x}} \cdot \underset{②}{\underline{x^2}} \, dx = \log x \cdot \frac{x^3}{3} - \int \left\{ \frac{1}{x} \times \frac{x^3}{3} \right\} \cdot dx$

$$\left[I_1 = \frac{\log x \cdot x^3}{3} - \frac{1}{3} \times \frac{x^3}{3} + c \right]$$

$I_2 = \int \log x \cdot 1 \cdot dx = \log x \cdot x - \int \frac{1}{\cancel{x}} \times \cancel{x} \cdot dx = \underline{\log x \cdot x - x}$

So: $I = I_1 + I_2$

$$= \frac{\log x \, x^3}{3} - \frac{x^3}{9} + \log x \cdot x - x + c$$

$$\int e^x \times \{ \underbrace{f(x)}_{1} + \underbrace{f'(x)}_{2} \} \cdot dx = e^x \cdot f(x) + c$$

Q. $\int e^x (\sin x + \cos x)\, dx = e^x \times \sin x + c$

Q. $\int \frac{x \cdot e^x}{(1+x)^2} \cdot dx = \int e^x \left\{ \frac{1+x}{(1+x)^2} - \frac{1}{(1+x^2)} \right\} \cdot dx$

Q. $\int x \cdot \sec^2 x \cdot dx$

Q. $\int \tan^{-1} x \cdot dx \Rightarrow \int \tan^{-1} x \times 1 \cdot dx$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ 1 $\quad$ 2

Q. $\int x\cdot(\log x)^2 = \int (\log x)^2 \times x\cdot dx = I$

$$I = (\log x)^2 \times \frac{x^2}{2} - \int \left\{ \cancel{2}\log x \times \frac{1}{\cancel{x}} \times \frac{x^{\cancel{2}}}{\cancel{2}} \right\} \cdot dx$$

$$I = \frac{(\log x)^2 \cdot x^2}{2} - \int \log x \cdot x \cdot dx \quad I_1$$

So $I_1 = \int \log x \cdot x \cdot dx = \log x \cdot \frac{x^2}{2} - \int \frac{1}{\cancel{x}} \times \frac{x^{\cancel{2}}}{2} \cdot dx = \left[\frac{\log x \cdot x^2}{2} - \frac{1}{2}\frac{x^2}{2} \right]$

So! $I = \frac{(\log x)^2 \cdot x^2}{2} - \frac{\log x \cdot x^2}{2} + \frac{x^2}{4} + c$

HW-

Q. $\int e^x \left\{\frac{1}{x} - \frac{1}{x^2}\right\} \cdot dx = e^x \times \frac{1}{x} + C$

Q. $\int \frac{e^x (x-3)}{(x-1)^3} \cdot dx = \int e^x \left\{ \frac{x-3}{(x-1)^3} \right\} \cdot dx$

$\rightarrow \int e^x \left\{ \frac{\cancel{x-1}}{(x-1)^3} - \frac{2}{(x-1)^3} \right\} \cdot dx$

$\rightarrow \int e^x \left\{ \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right\} dx$

let $\rightarrow \frac{1}{(x-1)^2} = f(x) \rightarrow f(x) = (x-1)^{-2}$

$f'(x) = -2(x-1)^{-2-1}$

$\boxed{f'(x) = \frac{-2}{(x-1)^3}}$

So :- $\int e^x \{f(x) + f'(x)\} \cdot dx$

$= e^x \times \left(\frac{1}{(x-1)^2}\right) + C$

Q. $\int \sin^{-1}\left\{\frac{2x}{1+x^2}\right\} \cdot dx$

$\Rightarrow$ let $x = \tan\theta$

derivate! $\rightarrow dx = \sec^2\theta \cdot \times d\theta$

$\rightarrow \int \sin^{-1}\left\{\frac{2\tan\theta}{1+\tan^2\theta}\right\} \times \sec^2\theta \times d\theta$

$\rightarrow \int \cancel{\sin^{-1}}\left\{\cancel{\sin}\, 2\theta\right\} \times \sec^2\theta \times d\theta$

$\Rightarrow 2\int \underbrace{\theta}_{\mathrm{I}} \cdot \underbrace{\sec^2\theta}_{\mathrm{II}} \cdot d\theta$