Ques:- $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} \, dx$

Divide:-

$$x^4+7x^2+12 \,\overline{)\, x^4+3x^2+2 \,}\, 1$$
$$\underline{-x^4 \pm 7x^2 + 12}$$
$$-4x^2-10$$

Sol. $I = \int \left[\overset{I_1}{1} + \overset{I_2}{\frac{(-4x^2-10)}{x^4+7x^2+12}} \right] dx$

$\Rightarrow I_2 = \int - \frac{(4x^2+10)}{x^4+7x^2+12} \, dx$

$$\frac{4x^2+10}{x^4+4x^2+3x^2+12} = \frac{4x^2+10}{x^2(x^2+4)+3(x^2+4)}$$

$$\Rightarrow \frac{4x^2+10}{(x^2+4)(x^2+3)} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{(x^2+3)}$$

$\Rightarrow 4x^2+10 = (Ax+B)(x^2+3) + (Cx+D)(x^2+4)$

$\Rightarrow 4x^2+10 = Ax^3 + 3Ax + Bx^2 + 3B + Cx^3 + 4Cx + Dx^2 + 4D$

Got →

$\boxed{10 = 3B + 4D}$ | $\boxed{4 = B + D}$ $\times 3$ | $\boxed{0 = 3A + 4C}$ | $\boxed{A + C = 0}$

$12 = 3B + 3D$

$-2 = D$

$4 = B + (-2)$

$\boxed{B = 6}$

$0 = 3(-C) + 4C$

$A = -C$

$\Rightarrow C = 0$

$A = 0$

$$I_2 = \int \frac{6}{x^2+4}\,dx + \int \frac{-2}{x^2+3}\,dx$$

$$I_2 = \cancel{6}\times\frac{1}{2}\tan^{-1}\frac{x}{2} - 2\cdot\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}$$

So : $I = I_1 - I_2$

$$I = x - 3\tan^{-1}\frac{x}{2} + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + c \;\; \text{Ans}$$

H.W

$$\int \frac{\cos x \cdot dx}{(1-\sin x)(2-\sin x)}$$

$\sin x \to t$

$\cos x \cdot dx = dt$

$$\Rightarrow \boxed{\int \frac{dt}{(1-t)(2-t)}}$$

Q. $\int \frac{2x}{(x^2+1)(x^2+3)} \cdot dx$

$x^2 = t$

$x \cdot dx = dt/2$

$\int \frac{dt/2}{(t+1)(t+3)}$

Q. $\int \frac{1}{(e^x-1)} \cdot dx \rightarrow$

$\int \frac{1}{(t-1)} \frac{dt}{t}$

let $e^x = t$

Deri. $\rightarrow$ $e^x \cdot \boxed{dx = dt}$

$dx = dt/e^x$

$dx = \frac{dt}{t}$

$\left[\frac{A}{(t-1)} + \frac{B}{t} \right]$

$$\int \underset{1}{u} \cdot \underset{2}{v} \, dx = u \cdot \int v \cdot dx - \int \left\{ \frac{d}{dx}(u) \times \int v \cdot dx \right\} \cdot dx$$

I L A T E

Inverse.

Log.

Arthe.

Trigo.

Expo.

$$\int \underset{1}{x} \cdot \underset{2}{\sin x}$$

$$\int \log x \cdot e^x$$

Q. $\int \underbrace{x}_{1} \cdot \underbrace{\sin x}_{2} \cdot dx$

So: $\int x \cdot \sin x \, dx = x \cdot \int \sin x \cdot dx - \int \left\{ \frac{d}{dx}(x) \times \int \sin x \cdot dx \right\} \cdot dx$

$$= x \times (-\cos x) - \int \{ 1 \times (-\cos x) \} \cdot dx$$

$$= -x\cos x + \int \cos x \cdot dx$$

$$= -x\cos x + \sin x + C$$

H.W. Q. $\int x \cdot \sin 3x \cdot dx$

H.W. Q. $\int x^2 \cdot e^x \cdot dx$

Q. $\int \underset{2}{x} \cdot \underset{\textcircled{1}}{\log 2x} \cdot dx = \int \log 2x \cdot x \cdot dx$

$\therefore \int \log 2x \cdot x \cdot dx = \log 2x \cdot \int x \cdot dx - \int \left\{ \frac{d}{dx}(\log 2x) \times \int x \cdot dx \right\} \cdot dx$

$= \log 2x \cdot \frac{x^2}{2} - \int \left\{ \frac{1}{\cancel{2}\cancel{x}} \times \cancel{2} \times \frac{\cancel{x^2}}{2} \right\} \cdot dx$

$= \frac{\log 2x \cdot x^2}{2} - \int \frac{x}{2} \cdot dx = \frac{\log 2x \cdot x^2}{2} - \frac{1}{2} \frac{x^2}{2} + C$ Ans

a. $\int x \cdot \sin^{-1} x \cdot dx = \int \sin^{-1} x \cdot x \cdot dx$ $\left[\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

(1) (2)

$\Rightarrow \int \sin^{-1} x \cdot x \cdot dx = \sin^{-1} x \cdot \frac{x^2}{2} - \int \left\{ \frac{1}{\sqrt{1-x^2}} \times \frac{x^2}{2} \right\} dx = I$

I_1

$\therefore I_1 = \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\, dx$

$\Rightarrow \frac{x^2}{\sqrt{1-x^2}} = \frac{-(1-x^2)}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$

So $I_1 = \frac{1}{2}\left[\int \frac{-(1-x^2)}{\sqrt{1-x^2}}\, dx + \int \frac{1}{\sqrt{1-x^2}}\, dx\right]$

$I_1 = \frac{1}{2}\left[-\int \sqrt{1-x^2}\, dx + \sin^{-1} x\right]$

$I_1 = \frac{1}{2}\left[-\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right]\right] + \frac{\sin^{-1} x}{2}$

So $I = \frac{\sin^{-1} x \cdot x^2}{2} + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1} x - \frac{\sin^{-1} x}{2} + c$

$$\int \frac{1}{x^2+a^2} \qquad \int \frac{1}{\sqrt{x^2+a^2}} \qquad \int \sqrt{a^2+x^2}$$

$$\int \frac{1}{x^2-a^2} \qquad \int \frac{1}{\sqrt{x^2-a^2}} \qquad \int \sqrt{x^2-a^2}$$

$$\int \frac{1}{a^2-x^2} \qquad \int \frac{1}{\sqrt{a^2-x^2}} \qquad \int \sqrt{a^2-x^2}$$