Neu < Den.

① $\dfrac{1}{(x+a)(x+b)} = \dfrac{A}{(x+a)} + \dfrac{B}{(x+b)}$

② $\dfrac{1}{(x+a)^2(x+b)} = \dfrac{A}{(x+a)} + \dfrac{B}{(x+a)^2} + \dfrac{C}{(x+b)}$

③ $\dfrac{1}{(x^2+a)(x+b)} = \dfrac{Ax+B}{(x^2+a)} + \dfrac{C}{x+b}$

$2 + \dfrac{1}{2} = \dfrac{5}{2}$

$\boxed{\text{Neu power} > \text{Den. power}}$ → Divide Neu. by Den.

→ after that ___ → $\dfrac{\text{Remi}}{\text{Den.}}$

$2 \overline{)\,5\,}\; 2$

$\quad 4$

$\quad \overline{1}$

Q. $\int \frac{x}{(x-1)^2(x+2)} \cdot dx \Rightarrow \frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$ — (1)

$$\frac{(x-1)^{-2+1}}{-2+1}$$

$\Rightarrow \frac{x}{(x-1)^2(x+2)} = \frac{A(x-1)(x+2) + B(x+2) + C(x-1)^2}{(x-1)^2(x+2)}$

$\Rightarrow x = Ax^2 + Ax - 2A + Bx + 2B + Cx^2 + C - 2Cx$

$\Rightarrow$ Const $\rightarrow$

$\boxed{0 = -2A + 2B + C}$

$\Rightarrow 0 = -2(-C) + 2B + C$

$\Rightarrow 0 = 3C + 2B$ — (ii)

$\Rightarrow 1 = -3C + B$

$1 = 3B$

$\Rightarrow B = \frac{1}{3}$

x-coeff:

$\boxed{1 = A + B - 2C}$

$\Rightarrow 1 = -C + B - 2C$

$\Rightarrow 1 = B - 3C$ — (iii)

$\Rightarrow 1 = \frac{1}{3} - 3C$

$\Rightarrow 3C = -\frac{2}{3}$

$C = -\frac{2}{9}$

x^2-coeff

$\boxed{0 = A + C}$

$A = -C$

$\Rightarrow A = \frac{2}{9}$

From eq (1)

$\Rightarrow \int \frac{2/9 \, dx}{(x-1)} + \int \frac{1/3 \, dx}{(x-1)^2} + \int \frac{-2/9}{(x+2)} \cdot dx$

$\frac{2}{9} \log|x-1| - \frac{2}{9} \log|x+2|$

$+ C$ Ans

Q. $\int \frac{3x+5}{x^3-x^2-x+1}\,dx = \frac{3x+5}{x^2(x-1)-1(x-1)} = \boxed{\frac{3x+5}{(x-1)(x^2-1)}} = \boxed{\frac{3x+5}{(x-1)(x-1)(x+1)}}$

$= \boxed{\frac{3x+5}{(x-1)^2(x+1)}} \longrightarrow$ 2nd

$$\int \frac{2x-3}{(x^2-1)(2x+3)} \cdot dx = \frac{2x-3}{(x-1)(x+1)(2x+3)}$$

Q. $\int \frac{x^3+x+1}{x^2-1}\,dx \rightarrow$

Divide Neu by Den.

$$x^2-1 \overline{)\, x^3 + x + 1 \,}\; x$$
$$\underline{\begin{array}{r} -x^3 \quad \underset{+}{-}x \end{array}}$$
$$2x+1$$

$\Rightarrow$ So! $\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$

$I = \underbrace{\int x\,dx}_{I_1} + \underbrace{\int \frac{2x+1}{x^2-1}\,dx}_{I_2}$

Nen. Power > Den. Power

So $I_2 = \int \frac{2x+1}{(x^2-1)}\,dx = \frac{2x+1}{(x-1)(x+1)}$

$\frac{2x+1}{(x-1)(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)} =$

$\Rightarrow 2x+1 = A(x+1) + B(x-1)$

$\Rightarrow \boxed{2 = A+B} \quad \Big| \quad \boxed{1 = A-B}$

$1 = A - B$

$3 = 2A \Rightarrow A = 3/2$

$1 = 3/2 - B$

$B = 1/2$

$I_2 = \int \frac{3/2}{(x-1)}\,dx + \frac{1/2}{(x+1)}\,dx =$

$I = \frac{x^2}{2} + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$

Q. $\int \frac{2}{(1-x)(1+x^2)} \cdot dx \quad \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)} = \frac{Ax+B}{(1+x^2)} + \frac{C}{(1-x)}$

Q.

Q. $\displaystyle\int \frac{1-x^2}{x(1-2x)}\,dx = \int \frac{1-x^2}{x-2x^2}\,dx$

Divide

$$x-2x^2 \,\big)\, 1-x^2 \,\big(\, \tfrac{1}{2}$$
$$\underline{\tfrac{x}{2} - x^2}$$
$$\;\; - \qquad +$$
$$-\tfrac{x}{2}+1$$

$$= \int \frac{1-x^2}{x-2x^2} = \frac{1}{2} + \frac{-\frac{x}{2}+1}{x-2x^2}$$

So! $I = \displaystyle\int \frac{1}{2}\,dx + \frac{1}{2}\int \frac{2-x}{x-2x^2}\,dx$

I_1 I_2

op $I_2 = \dfrac{1}{2}\displaystyle\int \frac{2-x}{x-2x^2}\,dx$

$$\frac{2-x}{x-2x^2} = \frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$$

$\Rightarrow 2-x = A(1-2x) + Bx$

$\Rightarrow \boxed{2 = A}$ $\Big|$ $-1 = -2A + B$

$-1 = -4 + B \Rightarrow \boxed{B = 3}$

$I_2 = \dfrac{1}{2}\left[\displaystyle\int \frac{2}{x}\,dx + \int \frac{3}{(1-2x)}\,dx\right]$

$I_2 = \dfrac{1}{2}\left[2\log x + 3\,\dfrac{\log|(1-2x)|}{-2}\right]$

So $I = \dfrac{x}{2} + \log|x| - \dfrac{3}{4}\log|1-2x| + c$

H.W

$$\int \frac{1}{x^4-1}\,dx = \frac{1}{(x^2)^2-(1)^2} = \frac{1}{(x^2-1)(x^2+1)} \qquad \frac{Ax+B}{(x^2-1)} + \frac{Cx+D}{(x^2+1)}$$

$$= \frac{1}{(x-1)(x+1)(x^2+1)}$$

$$\downarrow$$

$$\int \frac{A}{(x-1)} + \int \frac{B}{(x+1)} + \int \frac{Cx+D}{(x^2+1)}$$

Q. $\displaystyle\int \frac{1}{x(x^n+1)}\,dx =$

H.W Q. $\displaystyle\int \frac{x^3}{x^4(x^4-1)}\,dx$

$\Rightarrow \displaystyle\int \frac{dt/3}{t(t-1)}$

$\Rightarrow$ multiply & Divide by x^{n-1}

$\Rightarrow \displaystyle\int \frac{1 \times x^{n-1}}{x \cdot x^{n-1}(x^n+1)}\,dx = \int \frac{\boxed{x^{n-1}\,dx}}{x^n(x^n+1)}$

let $x^n = t \Rightarrow \boxed{n x^{n-1}\,dx = dt} \Rightarrow dt/n$

so: $\displaystyle\int \frac{x^{n-1}}{x^n(x^n+1)} = \int \frac{dt/n}{t(t+1)} = \frac{1}{n}\int \frac{1}{t(t+1)}\,dx$

$\Rightarrow \displaystyle\frac{1}{n}\left[\frac{A}{t} + \frac{B}{t+1}\right]$

H.W