Q. $\int \frac{x+2}{\sqrt{4x-x^2}}\,dx$

$\Rightarrow \int \frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}\,dx = \frac{-1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}\,dx + \int \frac{4}{\sqrt{4x-x^2}}\,dx$

I_1 I_2

Soln:- $(x+2) = A \times \frac{d}{dx}(4x-x^2) + B$

$\Rightarrow x+2 = A(4-2x)+B$

$x+2 = 4A - 2Ax + B$

Comparing $\Rightarrow$

$x \to$ coeff.

$2 = 4A + B$

$1 = -2A$

$A = -\frac{1}{2}$

$2 = 4\left(-\frac{1}{2}\right) + B$

$B = 4$

So $\left[x+2 = \frac{-1}{2}(4-2x)+4\right]$

$\therefore I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}\,dx$ $\quad 4x-x^2 \to t$

$I_1 = \int \frac{1}{\sqrt{t}}\,dt = 2\sqrt{t} = \boxed{2\sqrt{4x-x^2}} = I_1$

$I_2 = \int \frac{1}{\sqrt{4x-x^2}}\,dx \Rightarrow 4x-x^2$

$\Rightarrow -(x^2-4x)$

$\Rightarrow -\left[\underbrace{x^2 - 2\times x\times 2 + (2)^2}_{a^2 - 2\times a\times b} - (2)^2\right]$

$\Rightarrow -\left[(x-2)^2 - (2)^2\right] \Rightarrow \boxed{(2)^2 - (x-2)^2}$

So $I_2 = \int \frac{1}{\sqrt{(2)^2-(x-2)^2}}\,dx = \boxed{\sin^{-1}\frac{(x-2)}{2}} = I_2$

H.W

(1) $\int \frac{x+2}{\sqrt{x^2+2x+3}}\,dx$

$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}\,dx$

$= \frac{1}{2}\left[\int \frac{2x+2}{\sqrt{x^2+2x+3}}\,dx + \int \frac{2}{\sqrt{x^2+2x+3}}\,dx\right]$

$= \frac{1}{2}\left[I + I\right]$

Q. $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$ $\Rightarrow$ [Num. $= A \cdot \frac{d}{dx}(\text{Den.}) + B$]

$\Rightarrow 5x+3 = A \times (2x+4) + B$

$\Rightarrow 5 = 2A$ | $3 = 4A + B$

$A = 5/2$ | $3 = 2 \times \frac{5}{2} + B$

$B = -7$

So: $5x+3 = \frac{5}{2}(2x+4) + (-7)$

$\Rightarrow \int \frac{\frac{5}{2}(2x+4)}{\sqrt{x^2+4x+10}}\,dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}\,dx$

I_1 I_2

Q. $\int \frac{x}{(x+1)(x+2)} \cdot dx$

$$\Rightarrow \frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$

(1) (3)

$\Rightarrow x = Ax + 2A + Bx + B$

$\Rightarrow 0 = 2A + B$ | $1 = A + B$

$B = -2A$ | $1 = A + (-2A)$

$B = 2$ | $A = -1$

$\Rightarrow$ So: $\frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{x+2}$

$\Rightarrow \int \frac{-1}{x+1} \cdot dx + \int \frac{2}{x+2} \cdot dx$

$\Rightarrow -1 \log|x+1| + 2\log|x+2| + C$ Ans

(1) $\frac{1}{(x-a)(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-b)}$

(2) $\frac{1}{(x-a)(x-a)^2} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2}$

(3) $\frac{1}{(x-a)(x-b)^2} = \frac{A}{(x-a)} + \frac{Bx+C}{(x-b)^2}$

(4) $\frac{1}{(x-a)^2(x-b)} = \frac{Ax+B}{(x-a)^2} + \frac{C}{(x-b)}$

Q. $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$

Soln:-

$$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$$\Rightarrow 3x-1 = Ax^2-5Ax+6A+Bx^2-4Bx+3B+Cx^2-3Cx+2C$$

$\Rightarrow$ Comparing

$\Rightarrow -1 = 6A+3B+2C$

$\Rightarrow -1 = 6(-B-C)+3B+2C$

$\Rightarrow -1 = -6B-6C+3B+2C$

$\Rightarrow -1 = -3B-4C$

$\Rightarrow 3 = 3B+6C$

$8 = 2C$

$C = 4$

x-Coeff.

$3 = -5A-4B-3C$

$3 = -5(-B-C)-4B-3C$

$3 = 5B+5C-4B-3C$

$3 = B+2C$ $\times 3$

$3 = B+2(4)$

$B = -5$

x^2-Coeff.

$0 = A+B+C$

$A = -(B+C)$

$A = -(-5+4) \Rightarrow A = 1$

$$\int \frac{1}{(x-1)}\,dx - \int \frac{5}{x-2}\,dx + \int \frac{4}{x-3}\,dx = \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

H.W.

$$\int \frac{2x}{x^2+3x+2}$$

(I) $(x)^2 + 2 \times x \times \frac{3}{2} + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2$

$\left(x+\frac{3}{2}\right)^2 -$

(II) $x^2+2x+x+2$

$x(x+2)+1(x+2)$

$(x+2)(x+1)$

$(x-a)(x-b)$

$(x-a)^2(x-a)$