Q. $\int \frac{x-1}{\sqrt{x^2-1}}\cdot dx = \int \frac{x \cdot dx}{\sqrt{x^2-1}} - \int \frac{1}{\sqrt{x^2-1}}\cdot dx$

$\frac{4,5,6}{4}$

t

$\boxed{2x\,dx = dt}$

$\Rightarrow \int \frac{1}{\sqrt{t}}\cdot \frac{dt}{2}$

formula 4

Q. $\int \frac{x^2}{\sqrt{x^6 + a^6}} \cdot dx$

$(x^3)^2 + (a^3)^2$

$x^3 \to t$

$\left[\int \frac{dt/3}{\sqrt{t^2 + (a^3)^2}} \right]$

Q $\int \frac{\sec^2 x \cdot dx}{\sqrt{\tan^2 x + 4}} = \left[\int \frac{dt}{\sqrt{t^2 + (2)^2}} \right]$

$\tan x \to t$

Q. $\int \frac{1}{\sqrt{x^2+2x+2}} \cdot dx.$

$$\int \frac{1}{\sqrt{x^2+a^2}} \cdot dx$$

$$(a)^2 + 2 \times a \times b$$

$\therefore$ $x^2 + 2x + 2 \Rightarrow \underbrace{(x)^2 + 2 \times x \times 1 + (1)^2}_{} + (1)^2$

$$\Rightarrow (x+1)^2 + (1)^2$$

$\Rightarrow \therefore \int \frac{1}{\sqrt{\underbrace{(x+1)}_{t}^2 + (1)^2}} \cdot dx \Rightarrow$ let $x+1 = t \Rightarrow$ $\int \frac{1}{\sqrt{t^2+1}} \cdot dx$

$\Rightarrow dx = dt$

$\Rightarrow \log \left| t + \sqrt{t^2+1} \right| + C$ Ans

Q. $\displaystyle\int \frac{1}{\sqrt{9x^2+6x+5}}\,dx \Rightarrow 9x^2+6x+5 = \underbrace{\underbrace{(3x)^2}_{a^2} + \underbrace{2\times 3x\times 1 + (1)^2}_{2\times a\times b + b^2}}_{(3x+1)^2} + (2)^2$

$$\rightarrow \int \frac{1}{\sqrt{\underbrace{(3x+1)}_{t}{}^2 + (2)^2}}\,dx$$

Q. $\displaystyle\int \frac{1}{\sqrt{7-6x-x^2}}\,dx$

$\because\ 7-6x-x^2 = -\left[x^2+6x-7\right]$

$= -\left[(x)^2 + 2\times x\times 3 + (3)^2 - (3)^2 - 7\right]$

2×9 $\quad b$

$\Rightarrow -\left[(x+3)^2 - 16\right] = -\left[(x+3)^2 - (4)^2\right]$

$\Rightarrow \left[(4)^2 - (x+3)^2\right]$

$\Rightarrow \displaystyle\int \frac{1}{\sqrt{(4)^2-(x+3)^2}}\,dx \Rightarrow\ = \sin^{-1}\frac{(x+3)}{4} + C$

$$\int \frac{1}{\sqrt{(x-1)(x-2)}}\,dx \Rightarrow (x-1)(x-2) = x^2 - 3x + 2$$

$$(x)^2 - 2 \times x \times \overset{b}{\frac{3}{2}} + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + 2$$

Q. $\int \frac{1}{\sqrt{(x-a)(x-b)}}\,dx$

$\Rightarrow \therefore (x-a)(x-b) \Rightarrow \log\left|x-\left(\frac{a+b}{2}\right)+\sqrt{(x-a)(x-b)}\right|+C$

$\Rightarrow x^2 - x(a+b) + ab$

$\Rightarrow (x)^2 - 2 \times x \times \left(\frac{a+b}{2}\right) + \left(\frac{a+b}{2}\right)^2 - \left(\frac{a+b}{2}\right)^2 + ab$

$\Rightarrow \left[x-\left(\frac{a+b}{2}\right)\right]^2 + ab - \left[\frac{a^2+b^2+2ab}{4}\right]$

$\Rightarrow \left[x-\left(\frac{a+b}{2}\right)\right]^2 + \left[\frac{4ab - a^2 - b^2 - 2ab}{4}\right] \Rightarrow \left[x-\left(\frac{a+b}{2}\right)\right]^2 - \left[\frac{a^2+b^2-2ab}{4}\right]$

$\Rightarrow \left[x-\left(\frac{a+b}{2}\right)\right]^2 - \left(\frac{a-b}{2}\right)^2 \Rightarrow$ So $\int \frac{1}{\sqrt{\left[x-\left(\frac{a+b}{2}\right)\right]^2 - \left(\frac{a-b}{2}\right)^2}}\,dx$

t

$\log\left|x+\sqrt{x^2-a^2}\right|$

$\Rightarrow \log\left|x-\left(\frac{a+b}{2}\right)+\sqrt{\left[x-\left(\frac{a+b}{2}\right)\right]^2-\left(\frac{a-b}{2}\right)^2}\right|+C$

$\Rightarrow I = \int \frac{5x-2}{1+2x+3x^2}\,dx$

$\therefore \boxed{5x-2 = A \times \frac{d}{dx}(1+2x+3x^2) + B}$ — (1)

$\boxed{Neu. = A \times \frac{d}{dx}(Deno) + B}$

$\Rightarrow 5x - 2 = A \times (2+6x) + B$

Compaire Coefficient of x & Constant term :-

$\Rightarrow 5 = 6A$ & $-2 = 2A + B$

$\boxed{A = 5/6}$ — (2) $-2 = 2 \times \frac{5}{6} + B$

$B = -2 - \frac{5}{3} = \boxed{\frac{-11}{3} = B}$ — (3)

Q. $\int \frac{4x+1}{\sqrt{2x^2+x-3}}\,dx$

form (1) (2) & (3)

$5x-2 = \frac{5}{6}(2+6x) + \left(\frac{-11}{3}\right)$

Sol $\int \frac{\frac{5}{6}(2+6x) - 11/3}{1+2x+3x^2}\,dx$

$$\Rightarrow \int \left(\frac{\frac{5}{6}(2+6x) - \frac{11}{3}}{1+2x+3x^2} \right) dx \Rightarrow \left[\frac{5}{6} \underbrace{\int \frac{2+6x}{1+2x+3x^2} \cdot dx}_{I_1 \; t} - \frac{11}{3} \underbrace{\int \frac{1}{1+2x+3x^2} \cdot dx}_{I_2} \right]$$

let

$$\Rightarrow I_1 = \frac{5}{6} \int \frac{1}{t} \cdot dt = \boxed{\frac{5}{6} \log |1+2x+3x^2| + C} \; \text{①}$$

$$\Rightarrow I_2 \Rightarrow \boxed{1+2x+3x^2} \Rightarrow 3\left[x^2 + \frac{2}{3}x + \frac{1}{3}\right]$$

$$\Rightarrow 3\left[(x)^2 + 2 \times x \times \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 + \frac{1}{3}\right] \Rightarrow 3\left[\left\{x + \left(\frac{1}{3}\right)\right\}^2 - \frac{1}{9} + \frac{1}{3}\right]$$

$$\Rightarrow 3\left[\left(x+\frac{1}{3}\right)^2 + \frac{2}{9}\right] \Rightarrow I_2 = -\frac{11}{3} \times \frac{1}{3} \int \frac{1}{\left(x+\frac{1}{3}\right)^2 + \left(\frac{\sqrt{2}}{3}\right)^2} \cdot dx$$

$$= \frac{-11}{9/3} \times \frac{1}{\sqrt{2}/3} \times \tan^{-1} \left(\frac{x+1/3}{\sqrt{2}/3}\right)$$

So $I = I_1 - I_2$

$$\Rightarrow I_1 + \frac{11}{\sqrt{2} \cdot 3} \cdot \tan^{-1} \frac{3x+1}{\sqrt{2}} + C$$

②

Q. $\boxed{\int \frac{x+2}{\sqrt{x^2-1}} \cdot dx} \Rightarrow \int \frac{x}{\underbrace{\sqrt{x^2-1}}_{t}} \cdot dx + \int \frac{2}{\sqrt{x^2-1}} \cdot dx$

$$\frac{1}{\sqrt{x^2-a^2}}$$

Q. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}} \cdot dx \Rightarrow \int \frac{6x+7}{\sqrt{x^2-9x+20}}$

$\Rightarrow \boxed{6x+7} = A \times \frac{d}{dx}(x^2-9x+20) + B$

$\Rightarrow 6x\boxed{+7} = A \times (2x-9) + B$

$\Rightarrow 6 = 2A$ & $7 = -9A + B$

$\boxed{A = 3}$ $\quad 7 = -27 + B$

$\boxed{B = 34}$

Sol $6x+7 = 3 \times (2x-9) + 34$

SO!- $\int \frac{3 \times (2x-9) + 34}{\sqrt{x^2-9x+20}} \cdot dx$

$\Rightarrow 3\int \frac{2x-9}{\sqrt{x^2-9x+20}} \cdot dx + 34\int \frac{1}{\sqrt{x^2-9x+20}} \cdot dx$

I_1 $\quad t$ $\quad I_2$

$\Rightarrow 3\int \frac{1}{\sqrt{t}} \cdot dt + I_2$

I_1

$-\frac{81}{4} + 20$

$-\frac{1}{4} = \left(\frac{1}{2}\right)^2$

$I_2 = \because x^2 - 9x + 20$

$\Rightarrow (x)^2 - 2 \times x \times \frac{9}{2} + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2 + 20$

$\Rightarrow \left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\Rightarrow I_2 = 34\int \frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} \cdot dx \rightarrow$ formula 4