Q. $\int \frac{1-\cos x}{1+\cos x}\, dx$ $\quad \left\{ \cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \right\}$

$\because$ $1+\cos x = 2\cos^2\theta/2$

& $1-\cos x = 2\sin^2\theta/2$

So: $\int \frac{\cancel{2}\sin^2 x/2}{\cancel{2}\cos^2 x/2}\, dx = \int \tan^2\frac{x}{2}\, dx = \int \left(\sec^2\frac{x}{2} - 1\right) dx$

$\rightarrow \int \sec^2\frac{x}{2}\, dx - \int 1 \cdot dx \Rightarrow \frac{\tan\frac{x}{2}}{1/2} - x + C \Rightarrow 2\tan\frac{x}{2} - x + C$ Ans

H.W

Q. $\int \frac{\cos x}{1+\cos x} \cdot dx$

$\left\{ \cos 2\theta = \cos^2\theta - \sin^2\theta \right\}$

$\because \quad 1+\cos x = 2\cos^2\frac{x}{2}$

$\because \quad \cos x = \cos^2 x/2 - \sin^2 x/2$

$$\frac{\cos^2 x/2 - \sin^2 x/2}{2\cos^2 x/2}$$

Q. $\int \sin^4 x \, dx$

$$\left\{ \cos 2\theta = 1 - 2\sin^2\theta \right.$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

Solⁿ:- $\int \sin^2 x \cdot \sin^2 x \cdot dx$

$\because$ $\sin^2 x \cdot \sin^2 x = \left(\frac{1 - \cos 2x}{2}\right)^2 = \frac{1}{4}\left[1 + \cos^2 2x - 2\cos 2x\right]$

$\because$ $\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow \frac{\cos 2\theta + 1}{2} = \cos^2\theta$

So:- $\frac{1}{4}\left[1 + \left(\frac{\cos 4x + 1}{2}\right) - 2\cos 2x\right]$

So $\rightarrow \int \frac{1}{4}\left[1 + \frac{\cos 4x + 1}{2} - 2\cos 2x\right] \cdot dx \Rightarrow \frac{1}{4}\left[x + \frac{\sin 4x}{2 \times 4} + \frac{1}{2}x - \cancel{2}\frac{\sin 2x}{\cancel{2}} + C\right]$

H.W $\int \cos^4 2x \, dx$

$(\cos^2 2x)^2 = \left[\frac{1+\cos 2(2x)}{2}\right]^2$

$\cos^2 4x$

formula

Q. $\int \frac{\sin^2 x}{1+\cos x} \cdot dx$

$\because \sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\sin\theta = 2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$

$\because \sin^2\theta = 4 \cdot \sin^2\frac{\theta}{2} \cdot \cos^2\frac{\theta}{2}$

$\because \cos 2\theta = 2\cos^2\theta - 1$

$1 + \cos 2\theta = 2\cos^2\theta$

$1 + \cos\theta = 2\cos^2\theta/2$

Intg. $\rightarrow$ $\boxed{x - \sin x + C}$ Ans

So: $\int \frac{\sin^2 x}{2\cos^2 x/2} \cdot dx$

$\Rightarrow \frac{1}{\cancel{2}} \int \frac{\cancel{4} \cdot \sin^2\frac{x}{2} \cdot \cancel{\cos^2\frac{x}{2}}}{\cancel{\cos^2\frac{x}{2}}} \cdot dx$

$\rightarrow \int 2\sin^2\frac{x}{2} \cdot dx = \int 1 - \cos x \cdot dx =$

II $\rightarrow \int \frac{(1)^2 - \cos^2 x}{1+\cos x} = \int \frac{(1-\cos x)\cancel{(1+\cos x)}}{\cancel{(1+\cos x)}} dx$

$\int (1 - \cos x) \cdot dx$

$= x - \sin x + C$ Ans

Q. $\int \frac{\cos x - \sin x}{1+\sin 2x} \cdot dx = \frac{\cos x - \sin x}{1 + 2\sin x \cdot \cos x}$

$\Rightarrow \frac{\cos x - \sin x}{\sin^2 x + \cos^2 x + 2\sin x \cdot \cos x} = \frac{\cos x - \sin x}{\underbrace{(\sin x + \cos x)^2}_{t}}$

$\Rightarrow \int \frac{1}{t^2} \cdot dt \Rightarrow \int t^{-2} \cdot dt = \frac{t^{-2+1}}{-2+1} + c$

$\Rightarrow -\frac{1}{t} + c = -\frac{1}{(\sin x + \cos x)} + c$

Q. $\int \tan^{3} 2x \cdot \sec 2x \cdot dx \Rightarrow \int \tan^{2} 2x \cdot \tan 2x \cdot \sec 2x \cdot dx$

t $\quad t$

$dx = dt$

$\sec 2x \cdot \tan 2x$

$\Rightarrow \int (\sec^{2} 2x - 1) \boxed{\tan 2x \cdot \sec 2x \cdot dx}$

$\Rightarrow \sec 2x = t \Rightarrow \sec 2x \cdot \tan 2x \times 2 \cdot dx = dt$

$\Rightarrow \sec 2x \cdot \tan 2x \cdot dx = dt/2$

$\Rightarrow \int (t^{2} - 1) \cdot \frac{dt}{2} \Rightarrow \frac{1}{2} \left[\int t^{2} dt - \int 1 \cdot dt \right]$

$\Rightarrow \frac{1}{2} \left[\frac{t^{3}}{3} - t \right] + C \Rightarrow \frac{1}{2} \left[\frac{\sec^{3} 2x}{3} - \sec 2x \right] + C$ Ans

H.W

Q. $\int \tan^4 x \cdot dx$ $(\tan^2 x) \cdot (\tan^2 x)$

Q. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cdot \cos^2 x}$ → seperate

Q. $\int (\cos 2x + 2\sin^2 x) \cdot dx$

$\cos^2 x$