Ex:- $\int \frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}\,dx$

Solⁿ:- $\boxed{\tan^{-1} x^4 = t}$

$\Rightarrow \frac{1}{1+(x^4)^2} \cdot \times 4x^3 \cdot dx = dt \Rightarrow \boxed{\frac{x^3}{1+x^8} \cdot dx = \frac{dt}{4}}$

$\Rightarrow \int \sin t \frac{dt}{4} \Rightarrow \frac{-\cos t}{4} + C$

$\Rightarrow -\frac{\cos(\tan^{-1}x^4)}{4} + C$ Ans

Q. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} \cdot dx = ? = \log(x^{10} + 10^x) + C$

Q. $\int \frac{dx}{\sin^2 x \cdot \cos^2 x}$

$\int \frac{1}{\underline{\sin^2 x} \cdot \underline{\cos^2 x}} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} \cdot dx \Rightarrow \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$

$\rightarrow \int \sec^2 x \cdot dx + \int \operatorname{cosec}^2 x \cdot dx$

$\Rightarrow \tan x + (-\cot x) + C$

Q. $\int \sin^2(2x+5)\cdot dx.$

$\Rightarrow \cos 2\theta = 1 - 2\boxed{\sin^2\theta}$

$\Rightarrow \sin^2(2x+5) = \frac{1-\cos(4x+10)}{2}$

$\Rightarrow \int \frac{1-\cos(4x+10)}{2}\cdot dx = \int \frac{1}{2}\cdot dx - \int \frac{1}{2}\cos(4x+10)\cdot dx$

$\Rightarrow \frac{1}{2}x - \frac{1}{2}\frac{\sin(4x+10)}{4} + C$ Ans

Q. $\int (\sin 3x \cdot \cos 4x) \cdot dx$

Q. $\frac{2}{2}\int (\sin 3x \cdot \cos 3x)\, dx$

$\frac{1}{2}\int \sin 2(3x) \cdot dx$

$\Rightarrow \frac{1}{2} \frac{-\cos 6x}{6} + C$

Q. $\int (\sin 3x \cdot \cos 4x) \cdot dx$

$\because \sin A \cdot \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\Rightarrow \int \sin 3x \cdot \cos 4x \cdot dx = \int \frac{1}{2}\left[\sin 7x + \sin(-x)\right] dx$

$\Rightarrow \int \frac{1}{2} \sin 7x \cdot dx - \int \frac{1}{2} \sin x \cdot dx$

$\Rightarrow -\frac{1}{2} \frac{\cos 7x}{7} + \frac{1}{2} \cos x + C$ ✓

Q. $\int \left(\cos 2x \cdot \boxed{\cos 4x \cdot \cos 6x}\right) dx$

$\left\{ \cos 2\theta = 2\cos^2\theta - 1 \right\}$
$\cos^2\theta = \frac{1}{2}\left[\cos 2\theta + 1\right]$

$\Rightarrow \because \cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\therefore \cos 4x \cdot \cos 6x = \frac{1}{2}\left[\cos 10x + \cos(-2x)\right] = \frac{1}{2}\left[\cos 10x + \cos 2x\right]$

So! $\int \left[\cos 2x \cdot \frac{1}{2}\left(\cos 10x + \cos 2x\right)\right] \cdot dx = \frac{1}{2}\int \left(\cos 2x \cdot \cos 10x + \cos^2 2x\right) \cdot dx$

A B

$\Rightarrow \because \cos 2x \cdot \cos 10x = \frac{1}{2}\left(\cos 12x + \cos 8x\right)$ & $\cos^2 2x = \frac{1}{2}\left[\cos 4x + 1\right]$

$\Rightarrow \frac{1}{2}\int \frac{1}{2}\left(\cos 12x + \cos 8x\right) \cdot dx + \frac{1}{2}\int \frac{1}{2}\left(\cos 4x + 1\right) \cdot dx$

$\Rightarrow \frac{1}{2} \cdot \frac{1}{2}\left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8}\right] + \frac{1}{4}\left(\frac{\sin 4x}{4} + x\right) + C$ Ans

H.W Q. $\int \sin x \cdot \sin 2x \cdot \sin 3x = \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$

Q. $\int \sin 4x \cdot \sin 8x \cdot dx$

Q. $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} \cdot dx$

Q. $\int \sin^3(2x+1)\cdot dx = \int \sin^2(2x+1)\cdot \sin(2x+1)\cdot dx$

$\sin^2\theta = 1-\cos^2\theta \Rightarrow \sin^2(2x+1) = 1-\cos^2(2x+1)$

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$\Rightarrow \int (1-\cos^2(2x+1)\cdot \sin(2x+1)\cdot dx$

let $\cos(2x+1) = t$

diff $-\sin(2x+1)dx = -dt$

$\Rightarrow \left[\int \sin(2x+1)\cdot dx - \int \cos^2(2x+1)\cdot \sin(2x+1)\cdot dx\right]$

$\Rightarrow \left[\frac{-\cos(2x+1)}{2}\right] + \int t^2\cdot dt$

$= \quad + \frac{t^3}{3} + C$