

Ex: $\int \frac{(\log x)^2}{x} dx$

Sol: let $(\log x = t)$

Derivate: $\left[\frac{1}{x} dx = dt \right]$

$\Rightarrow \int t^2 dt = \frac{t^3}{3} + C$

$\Rightarrow \frac{(\log x)^3}{3} + C$

Integral

Ex: $\int x \sqrt{x+2} dx$

\Rightarrow let $(x+2) = t$ — (1)
 $\hookrightarrow (x = t-2)$ — (2)

\rightarrow Diff $\rightarrow [dx = dt]$ — (3)

$\Rightarrow \int (t-2) \sqrt{t} dt$

$\rightarrow \int t^{3/2} dt - \int 2 \cdot t^{1/2} dt$

Integ: $\Rightarrow t^{5/2} \cdot \frac{2}{5} - 2 \cdot t^{3/2} \cdot \frac{2}{3} + C$

$\Rightarrow \frac{2(x+2)^{5/2}}{5} - \frac{4}{3}(x+2)^{3/2} + C$

$\Rightarrow \frac{1}{4}(x^3-1)^{1/3} + \frac{1}{5}(x^3-1)^{5/3} + C$

Ex: $\int \frac{(x^3-1)^{1/3}}{x^5} dx$

$\Rightarrow \int \frac{(x^3-1)^{1/3}}{x^5} \cdot x^2 dx$

\rightarrow let $(x^3-1) = t$

diff $\rightarrow 3x^2 dx = dt$
 $\Rightarrow x^2 dx = \frac{dt}{3}$

$\Rightarrow \int t^{1/3} (t+1) \cdot \frac{dt}{3}$

$\Rightarrow \int t^{4/3} dt + \int t^{1/3} dt$

$\Rightarrow \frac{t^{7/3}}{7/3} + \frac{t^{4/3}}{4/3}$

\Rightarrow

Ex:

$$\int \frac{1}{x(\log x)^m} dx$$

Ex: $\int \frac{x \cdot dx}{e^{x^2}}$

Ex $\int \frac{(e^{2x}-1)}{e^{2x}+1} dx \rightarrow$ divide num & Den. by e^x .

Solⁿ: $\log x = t$
 $\frac{1}{x} \cdot dx = dt$

\Rightarrow let $x^2 = t$
 $\rightarrow 2x dx = dt$
 $\rightarrow x dx = dt/2$

$\Rightarrow \int \frac{\frac{e^{2x}-1}{e^x} - \frac{1}{e^x}}{\frac{e^{2x}}{e^x} + \frac{1}{e^x}} dx \Rightarrow \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$\Rightarrow \int \frac{1}{t^m} dt$

$\rightarrow \int \frac{1}{e^t} \cdot \frac{dt}{2} = \frac{1}{2} \int e^{-t} dt$

$\Rightarrow \int t^{-m} dt$

\Rightarrow let $(e^x + e^{-x}) = t$

Diff $\rightarrow (e^x + e^{-x}(-1)) dx = dt$

$\Rightarrow \frac{t^{-m+1}}{-m+1} + C$

$\Rightarrow \frac{1}{2} \cdot \frac{e^{-t}}{-1} + C$

Sol: $\int \frac{dt}{t} = \log t + C$

$\Rightarrow \frac{\log x^{1-m}}{1-m} + C$

$\Rightarrow -\frac{1}{2} e^{-x^2} + C$

$= -\frac{1}{2} \frac{1}{e^{x^2}} + C$

$= \log(e^x + e^{-x}) + C$

Ex: $\int \sec^2(7-4x) \cdot dx$

Solⁿ: $\Rightarrow \frac{\tan(7-4x) + C}{-4}$

\Rightarrow let $7-4x = t$
 $\Rightarrow 0-4(dx = dt)$
 $dx = \frac{dt}{-4}$

$\rightarrow \int \sec^2 t \cdot \frac{dt}{-4}$

$\Rightarrow \frac{-1}{4} \int \sec^2 t \cdot dt$

$\Rightarrow \frac{-1}{4} \times \tan t + C = \frac{-1}{4} \tan(7-4x) + C$

Ex: $\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} \cdot dx$

$\Rightarrow 6 \cos x + 4 \sin x = t$
 diff $\rightarrow (6 \sin x + 4 \cos x) dx = dt$

$\Rightarrow 2[2 \cos x - 3 \sin x] dx = dt$

So: $\int \frac{1}{t} \cdot \frac{dt}{2} = \frac{1}{2} \int \frac{1}{t} \cdot dt$

$\Rightarrow \frac{1}{2} \log t + C$

$= \frac{1}{2} \log(6 \cos x + 4 \sin x) + C$

Ex: $\int \cot x \cdot \log \sin x \cdot dx$

let $\log \sin x = t$

diff $\rightarrow \frac{1 \times \cos x \cdot dx}{\sin x} = dt$

$\rightarrow \cot x \cdot dx = dt$

$\rightarrow \int t \cdot dt$

$= \frac{t^2}{2} + C$

$\Rightarrow \frac{(\log \sin x)^2}{2} + C$

H.W. → ① $\int (4x+2)\sqrt{x^2+x+1} \cdot dx$

② $\int \frac{\cos \sqrt{x} \cdot dx}{\sqrt{x}}$

③ $\int \frac{x}{9-4x^2} \cdot dx$

④ $\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} \cdot dx$

Ex 1- $\int \frac{\sin x \cdot dx}{1 + \cos x} \rightarrow t$ $\left. \begin{array}{l} \sin x + \cos x = t \\ \text{diff} \rightarrow (\cos x - \sin x) dx = dt \\ \rightarrow -(\sin x + \cos x) dx = dt \end{array} \right\} \Rightarrow \frac{x}{2} - \frac{1}{2} \log t + C$

$\rightarrow \int \frac{-dt}{t} \rightarrow -\log(1 + \cos x) + C$ $\left. \begin{array}{l} \Rightarrow \frac{x}{2} - \frac{1}{2} \log(\sin x + \cos x) + C \end{array} \right\}$

Ex 2- $\int \frac{1}{1 + \cot x} dx$

$\rightarrow \int \frac{1}{1 + \frac{\cos x}{\sin x}} dx \rightarrow \frac{1}{2} \left[\int \frac{\sin x + \cos x}{\sin x + \cos x} + \int \frac{\sin x - \cos x}{\sin x + \cos x} \right] dx$

$\rightarrow \frac{1}{2} \left[\int \left(1 + \frac{\sin x - \cos x}{\sin x + \cos x} \right) dx \right]$

$\rightarrow \int \frac{\sin x \cdot dx}{\sin x + \cos x}$

$\rightarrow \frac{1}{2} \int \frac{2 \sin x}{\sin x + \cos x} dx$

$\Rightarrow \frac{1}{2} \int 1 \cdot dx + \frac{1}{2} \int \frac{\sin x - \cos x}{\sin x + \cos x} dx$

$\Rightarrow \frac{1}{2} x + \frac{1}{2} \int \frac{1}{t} (-dt) t$

Hw $\int \frac{1}{1 + \tan x} dx$