Integral

Ex:- $\int (4 \cdot e^{3x} + 1) \cdot dx$

Sol:- $\int 4 \cdot e^{3x} \cdot dx + \int 1 \cdot dx$

$\Rightarrow 4\int e^{3x} \cdot dx + \int 1 \cdot dx$

$\Rightarrow 4 \cdot \frac{e^{3x}}{3} + x + C$ Ans

Ex:- $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 \cdot dx$

Sol:- $\Rightarrow \int \left(x + \frac{1}{x} - 2 \cdot \sqrt{x} \times \frac{1}{\sqrt{x}}\right) \cdot dx$

$\Rightarrow \int x \cdot dx + \int \frac{1}{x} \cdot dx - \int 2 \cdot dx$

$\Rightarrow \frac{x^2}{2} + \log|x| - 2x + C$ Ans

Ex:- $\int \frac{x^3 - x^2 + x - 1}{x - 1} \cdot dx$

$\Rightarrow \int \frac{x^2(x-1) + (x-1)}{(x-1)} \cdot dx$

$\Rightarrow \int \frac{\cancel{(x-1)}(x^2+1)}{\cancel{(x-1)}} \cdot dx$

$\Rightarrow \int (x^2+1) \cdot dx$

$\Rightarrow \int x^2 \cdot dx + \int 1 \cdot dx$

$\Rightarrow \frac{x^{2+1}}{2+1} + x + C$

$\Rightarrow \frac{x^3}{3} + x + C$ Ans

Ex:- $\int \sec x(\sec x + \tan x) \cdot dx$

$\Rightarrow \int (\sec^2 x + \sec x \tan x) dx$

$\Rightarrow \int \sec^2 x \cdot dx + \int \sec x \tan x \, dx$

Integrate:-

$\Rightarrow \tan x + \sec x + c$

Ans

Integral

Ex:- $\int \frac{2-3\sin x}{\cos^2 x}\cdot dx$

Sol^n:-

$\int \frac{2}{\cos^2 x}\cdot dx - \int \frac{3\sin x}{\cos^2 x}\cdot dx$

$\Rightarrow 2\int \sec^2 x \cdot dx - 3\int \tan x\cdot \sec x\cdot dx$

$\Rightarrow 2\cdot \tan x - 3\sec x + c$ Ans

$\Rightarrow$ $f(x)$

$\downarrow$

Derivative $\rightarrow \frac{d}{dx}f(x)$

$\downarrow$

Integrate $\rightarrow \int \frac{d}{dx}\cdot f(x)\cdot dx \longrightarrow f(x)+c$

$\int x^n = \frac{x^{n+1}}{n+1}$

$16+\frac{1}{8}$

$=128+1$

Ex:- if $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2)=0$ then $f(x)=?$

Sol^n:- $\because \frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$

let's integrate the above fun.

$\Rightarrow \int \frac{d}{dx}f(x)\cdot dx = \int \left(4x^3 - \frac{3}{x^4}\right)\cdot dx$

$\Rightarrow f(x) = 4\cdot\int x^3\cdot dx - 3\int \frac{1}{x^4}\cdot dx$

$f(x) = 4\cdot\frac{x^4}{4} - 3\int x^{-4}\cdot dx$

$f(x) = x^4 - 3\cdot\frac{x^{-4+1}}{-4+1} + c$

$f(x) = x^4 - 3\cdot\frac{x^{-3}}{-3} + c = \boxed{x^4 + \frac{1}{x^3} + c = f(x)}$

$\because f(2) = 0 \Rightarrow (2)^4 + \frac{1}{(2)^3} + c = 0 \Rightarrow c = -\frac{129}{8}$

So! $f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$ Ans

Integral

Types/ Method of Integration.

① Substitution Method :-

Ex :- $\int \sin(mx)\, dx$

$I \rightarrow \dfrac{-\cos mx}{m} + c$ Ans

II → Sol → let $\boxed{mx = \theta}$ —①

diff ⇒ $m \cdot \dfrac{dx}{d\theta} = 1$

$m \cdot dx = d\theta$

$\left[dx = \dfrac{d\theta}{m}\right]$ —②

$\Rightarrow \int \sin\theta \cdot \dfrac{d\theta}{m} = \dfrac{1}{m}\int \sin\theta \cdot d\theta$

$\Rightarrow \dfrac{1}{m} - \cos\theta + c$

$\Rightarrow \dfrac{1}{m} - \cos mx + c$ Ans

$\boxed{= \dfrac{-\cos mx}{m} + c}$

H.W → $\int \cot x \cdot dx$

H.W → $\int x \cdot \sqrt{x+2} \cdot dx$

Ex: $\int \dfrac{\boxed{2x \cdot dx}}{1+x^2}$

⇒ let $1+x^2 = t$

diff → $0 + \dfrac{\boxed{2x \cdot dx}}{dt} = 1$

$\Rightarrow 2x \cdot dx = dt$

$\Rightarrow \int \dfrac{1}{t} \cdot dt$

$\Rightarrow \log|t| + c$

$\Rightarrow \log|1+x^2| + c$ Ans

Ex :- $\int \sin^3 x \cdot \cos^2 x \cdot dx$

$\Rightarrow \int \sin^2 x\, \boxed{\sin x} \cdot \cos^2 x\, \boxed{dx}$

$\cos x = t$

$-\sin x \cdot dx = dt$

$\sin x \cdot dx = -dt$

$\Rightarrow \int (1-\cos^2 x) \cdot \cos^2 x \cdot \sin x \cdot dx$

$\Rightarrow \int (1-t^2) \cdot t^2 \cdot (-dt) \Rightarrow -\int (t^2 - t^4) \cdot dt \Rightarrow -\int t^2 dt + \int t^4 dt$

$\Rightarrow -\dfrac{t^3}{3} + \dfrac{t^5}{5} + c$ Ans $\Rightarrow -\dfrac{\cos^3 x}{3} + \dfrac{\cos^5 x}{5} + c$ Ans