Q. The function f(x) = $2x^3 - 3x^2 - 12x + 4$, has

(a) two points of local maximum

(b) two points of local minimum

(c) one maxima and one minima

(d) no maxima or minima

Sol^n:- $f'(x) = 6x^2 - 6x - 12$

$= 6[x^2 - x - 2]$

$= 6[x^2 - 2x + x - 2]$

$= 6[x(x-2) + 1(x-2)]$

$= 6(x+1)(x-2)$

let $f'(x) = 0 \Rightarrow \boxed{x = -1, 2}$

$\Rightarrow \boxed{f''(x) = 12x - 6}$

$x = -1 \rightarrow f''(-1) = -12 - 6 = -18 < 0$

$\rightarrow$ point of max.

$x = 2 \rightarrow f''(2) = 24 - 6 = 18 > 0$

$\rightarrow$ point of min

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The smallest value of the polynomial

$x^3 - 18x^2 + 96x$ in [0, 9] is

(a) 126 $x = \boxed{4, 8}$ (b) 0

(c) 135 $0, 9$ (d) 160

0

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If the function f be given by

$f(x) = x^3 - 3x + 3$, then $\rightarrow f'(x) = 3x^2 - 3$ — (1)

I. $x = \pm 2$ are the only critical points for local maxima or local minima.

II. $x = 1$ is a point of local minima.

III. local minimum value is 2.

IV. local maximum value is 5.

(a) Only I and II are true

(b) Only II and III are true

(c) Only I, II and III are true

(d) Only II and IV are true

let $f'(x) = 0$

$3x^2 - 3 = 0 \Rightarrow x^2 = 1 \Rightarrow \boxed{x = \pm 1}$

from (1):-

$f''(x) = 6x$

point of mini $\rightarrow$ at $x = 1 \rightarrow f''(x) = 6 > 0$

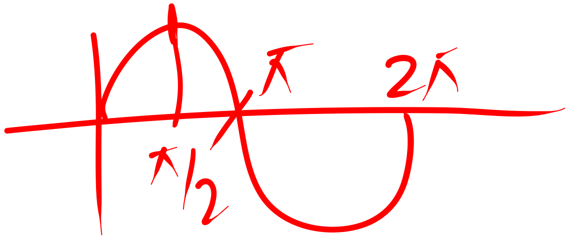
$\rightarrow f(1) = (1)^3 - 3(1) + 3 = 1$

point of max. $\rightarrow$ at $x = -1 \rightarrow f''(-1) = -6 < 0$

$\rightarrow f(-1) = (-1)^3 - 3(-1) + 3 = 5$

Assertion: The maximum value of the function y = sin x in [0, 2π] is at $x = \frac{\pi}{2}$.

Reason: The first derivative of the function is zero at $x = \frac{\pi}{2}$ and second derivative is negative at $x = \frac{\pi}{2}$.



$y' = \cos x =$

$y'' = -\sin x = 1$

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion

(c) Assertion is correct, Reason is incorrect

(d) Assertion is incorrect, Reason is correct.

There is an error of 0.04 cm in the measurement of the diameter of a sphere. When the radius is 10 cm, the percentage error in the volume of the sphere is

(a) ± 1.2 (b) ± 1.0

(c) ± 0.6 (d) ± 0.8

$r = 0.02 \times 3$

0.06×10

$= \boxed{\pm 0.6}$

If $x = \sqrt{a^{\sin^{-1}t}}$ and $y = \sqrt{a^{\cos^{-1}t}}$, then

(a) $x\dfrac{dy}{dx} + y = 0$ (b) $x\dfrac{dy}{dx} = y$

(c) $y\dfrac{dy}{dx} = x$ (d) None of these

Sol^n:

$x = \left(a^{\sin^{-1}t}\right)^{1/2} = a^{\frac{1}{2}\sin^{-1}t}$

$\rightarrow \dfrac{dx}{dt} = a^{\frac{1}{2}\sin^{-1}t} \cdot \log a \left(\dfrac{1}{2} \cdot \dfrac{1}{\sqrt{1-t^2}}\right)$

$\rightarrow \dfrac{dx}{dt} = \sqrt{a^{\sin^{-1}t}} \cdot \log a \cdot \left(\dfrac{1}{2\sqrt{1-t^2}}\right)$ — (1)

$\Rightarrow y = \left(a^{\cos^{-1}t}\right)^{1/2}$

$\Rightarrow y = a^{\frac{1}{2}\cos^{-1}t}$ — (11)

$\rightarrow \dfrac{dy}{dt} = a^{\frac{1}{2}\cos^{-1}t} \times \log a \times \left(\dfrac{1}{2} \cdot \dfrac{-1}{\sqrt{1-t^2}}\right)$

divide eq (11) by (1)

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-\sqrt{a^{\cos^{-1}t}} \times \cancel{\log a} \times \left(\cancel{\dfrac{1}{2\sqrt{1-t^2}}}\right)}{\sqrt{a^{\sin^{-1}t}} \times \cancel{\log a} \times \left(\cancel{\dfrac{1}{2\sqrt{1-t^2}}}\right)}$

$\boxed{\dfrac{dy}{dx} = -\dfrac{y}{x}}$ → (A)

$x \cdot \dfrac{dy}{dx} = -y \Rightarrow \boxed{x \cdot \dfrac{dy}{dx} + y = 0}$ (A)

Which of the following is/are true?

Statement I: If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, then $\boxed{\frac{dy}{dx}} = -\cot\frac{\theta}{2}$.

Statement II : If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

then derivative of y with respect to x is $-\cot 3t$.

(a) Only I is true.

(b) Only II is true.

(c) Both I and II are true.

(d) Neither I nor II is true.

$dy/dx =$

$\cos 2\theta$

$\sin 2\theta$

$\cos\theta$

$\sin\theta$

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If the function,

$$f(x)=\begin{cases} x+a^2\sqrt{2}\sin x & , \ 0\le x\le \pi/4 \\ x\cot x+b & , \ \pi/4\le x\le \pi/2 \\ b\sin 2x-a\cos 2x & , \ \pi/2\le x\le \pi \end{cases}$$

is continuous in the interval $[0,\pi]$ then the value of (a, b) are :

(a) $(-1,-1)$ (b) $(0,0)$

(c) $(-1,1)$ (d) $(1,0)$

$x=\pi/4$

LHL = RHL

$\lim_{x\to\pi/4^-} (x-h)+a^2\sqrt{2}\sin(x-h) = \lim_{x\to\pi/4^+} (x+h)\cot(x+h)+b$

$\Rightarrow \left(\frac{\pi}{4}-0\right)+a^2\sqrt{2}\sin\left(\frac{\pi}{4}-0\right) = \frac{\pi}{4}\cdot\cot\left(\frac{\pi}{4}+0\right)+b$

$= \frac{\pi}{4}+a^2 = \frac{\pi}{4}\cdot 1+b \Rightarrow a^2=b$ (1)

$(-1)^2=1 \to (-1,1)$

$x=\pi/2$

LHL = RHL $\to \lim_{x\to\pi/2^-} = \lim_{x\to\pi/2^+}$

$\Rightarrow \left(\frac{\pi}{2}-0\right)\cot\left(\frac{\pi}{2}-0\right)+b = b\sin 2\left(\frac{\pi}{2}+0\right) - a\cos 2\left(\frac{\pi}{2}+0\right)$

$\Rightarrow b=a$ — (2)

from eqn (1) & (2)

$\Rightarrow (0,0)$ or $(1,1)$

Let function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then f is

(a) one-one and onto

(b) one-one but NOT onto

(c) onto but NOT one-one

(d) neither one-one nor onto

Let $A = N \times N$ and * be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Then * is

(a) commutative (b) associative

(c) Both (a) and (b) (d) None of these

1	2	3	4	5	6	7	8	9	10
C	B	D	A	C	A	C	B	A	C

The difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{2}$

(b) π

(c) $\frac{3\pi}{2}$

(d) $\frac{\pi}{4}$

The radius of a sphere initially at zero increases at the rate of 5 cm/sec. Then its volume after 1 sec is increasing at the rate of :

(a) 50π (b) 5π

(c) 500π (d) None of these

The curve $y = x^{\frac{1}{5}}$ at (0, 0) has

(a) a vertical tangent (parallel to y-axis)

(b) a horizontal tangent (parallel to x-axis)

(c) no oblique tangent

(d) no tangent

The smallest value of the polynomial

$x^3 - 18x^2 + 96x$ in $[0, 9]$ is

(a) 126 (b) 0

(c) 135 (d) 160

A point c in the domain of a function f is called a critical point of f if

I. $f'(c) = 0$

II. f is not differentiable at c.

Choose the correct option

(a) Either I or II are true

(b) Only I is true

(c) Only II is true

(d) Neither I nor II is true

The function $f(x) = x^2 \log x$ in the interval $[1, e]$ has

(a) a point of maximum and minimum

(b) a point of maximum only

(c) no point of maximum and minimum in $[1, e]$

(d) no point of maximum and minimum

If $y = \dfrac{ax - b}{(x-1)(x-4)}$ has a turning point $P(2, -1)$, then the value of a and b respectively, are

(a) $1, 2$ (b) $2, 1$

(c) $0, 1$ (d) $1, 0$

Ques:

An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. The cost of the material will be least when depth of the tank is

(a) twice of its width

(b) half of the width

(c) equal to its width

(d) None of these

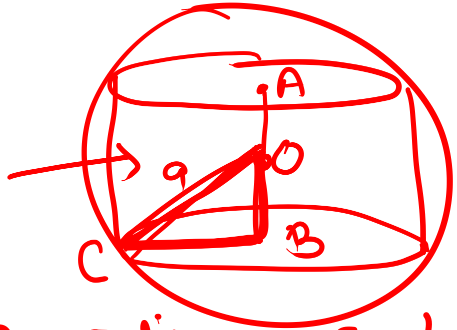
Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a.

(a) 2a/3

(b) $\frac{2a}{\sqrt{3}}$

(c) a/3

(d) a/5



Sol^n: Let a = radius of sphere

AB = Height of cyli = 2×OB = $\boxed{2x = h}$

Let $\boxed{OB = x}$ &

BC = radius of cylin = $r = \sqrt{a^2 - x^2}$

∵ vol. of cylin = $\pi r^2 h = \boxed{\pi(a^2 - x^2)2x = V} \to$ max.

$\pi(2xa^2 - 2x^3) = V$

$\Rightarrow V' = \pi[2a^2 - 6x^2] \Rightarrow$ Let $V' = 0$

$\Rightarrow 2a^2 - 6x^2 = 0 \Rightarrow 6x^2 = 2a^2$

$\Rightarrow x^2 = \frac{a^2}{3} \Rightarrow \boxed{x = \pm\frac{a}{\sqrt{3}}}$

diff again:

$V'' = \pi[0 - 12x] = -12x\pi$

find at $x = a/\sqrt{3} \to \boxed{V'' < 0} \to$ so vol is max.

so Height $h = 2x$

$\boxed{h = 2 \times \frac{a}{\sqrt{3}}}$

The maximum value of $\left(\frac{1}{x}\right)^x$ is y

(a) e

(b) e^e

(c) $e^{\frac{1}{e}}$

(d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

$\log y = \log\left(\frac{1}{x}\right)^x$

$\log y = x \cdot \log x^{-1}$

$\log y = [-x \cdot \log x]$

diff $\rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -\left[x \cdot \frac{1}{x} + \log x\right]$

$\boxed{\frac{dy}{dx} = -y[1+\log x]} \rightarrow$ Let $\frac{dy}{dx} = 0$

$\Rightarrow -y[1+\log x] = 0 \Rightarrow \boxed{\log x = -1}$

$\rightarrow \boxed{x = e^{-1} = 1/e}$ — (1)

So at $\boxed{x = 1/e}$

So max. value of $y = (e)^{1/e}$

$\Rightarrow$ diff again

$\frac{d^2y}{dx^2} = -\left[y\left(\frac{1}{x}\right) + (1+\log x)\frac{dy}{dx}\right]$

$\frac{d^2y}{dx^2} = -\left[\frac{y}{x} + (1+\log x)(-y)(1+\log x)\right]$

$\frac{d^2y}{dx^2} = -\frac{y}{x} + y(1+\log x)^2$

$\frac{d^2y}{dx^2} = y\left[-\frac{1}{x} + (1+\log x)^2\right]$

$\frac{d^2y}{dx^2} = \left(\frac{1}{x}\right)^x\left[-\frac{1}{1/e} + (1 + (-1))^2\right]$

$d^2y/dx^2 = (e)^{1/e}[-e] \rightarrow \boxed{<0}$

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1	2	3	4	5	6	7	8	9	10
B	C	B	B	A	C	D	B	B	C