ABLES KOTA

The difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{2}$

(b) π

(c) $\frac{3\pi}{2}$

(d) $\frac{\pi}{4}$

$0 + \pi/2$

$\sin \cancel{2}(\pi/3) - (+\pi/6)$

$f'(x) = 2\cos 2x - 1 = 0$ $\quad + \sin \pi/3 - \pi/6 = \frac{+\sqrt{3}}{2} - \frac{\pi}{6}$

$\Rightarrow \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3} = \cos(-\pi/3)$

$2x = \frac{\pi}{3}$ or $-\frac{\pi}{3}$

$\boxed{x = \pi/6}$ or $\boxed{(-\pi/6)}$

$\sin(\pi) - \pi/2$

$= 0 - \pi/2$

$f(-\pi/2) = \pi/2 \rightarrow$ greatest $\rightarrow$ max.

$f(-\pi/6) = \frac{-\sqrt{3}}{2} + \pi/6$

$f(\pi/6) = \frac{\sqrt{3}}{2} - \pi/6$

$f(\pi/2) = -\pi/2 \rightarrow$ least $\rightarrow$ min.

$\Rightarrow$ Diff $= \pi/2 - (-\pi/2) = \pi$

The radius of a sphere initially at zero increases at the rate of 5 cm/sec. Then its volume after 1 sec is increasing at the rate of :

(a) 50π (b) 5π

(c) 500π (d) None of these

$\frac{dr}{dt} = 5$ cm/sec

$V = \frac{4}{3}\pi r^3$

Change in vol.

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \times \cancel{3} r^2 \times \frac{dr}{dt}$

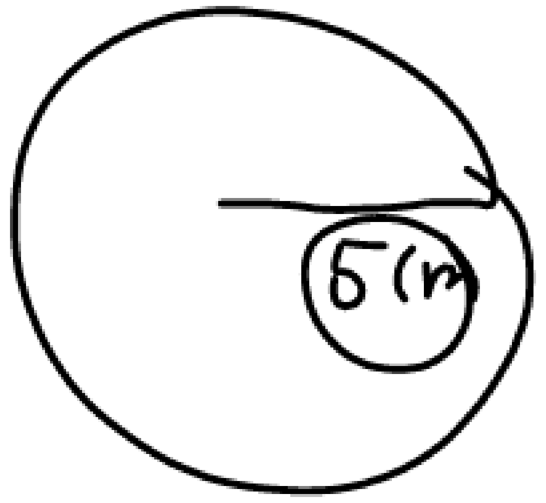
So vol. after 1 sec is:

$\left.\frac{dV}{dt}\right|_{r=5} = 4\pi(5)^2 \times 5 = 4\pi r^2 \times \frac{dr}{dt}$

$= 500\pi$

0 sec → 0

1 sec



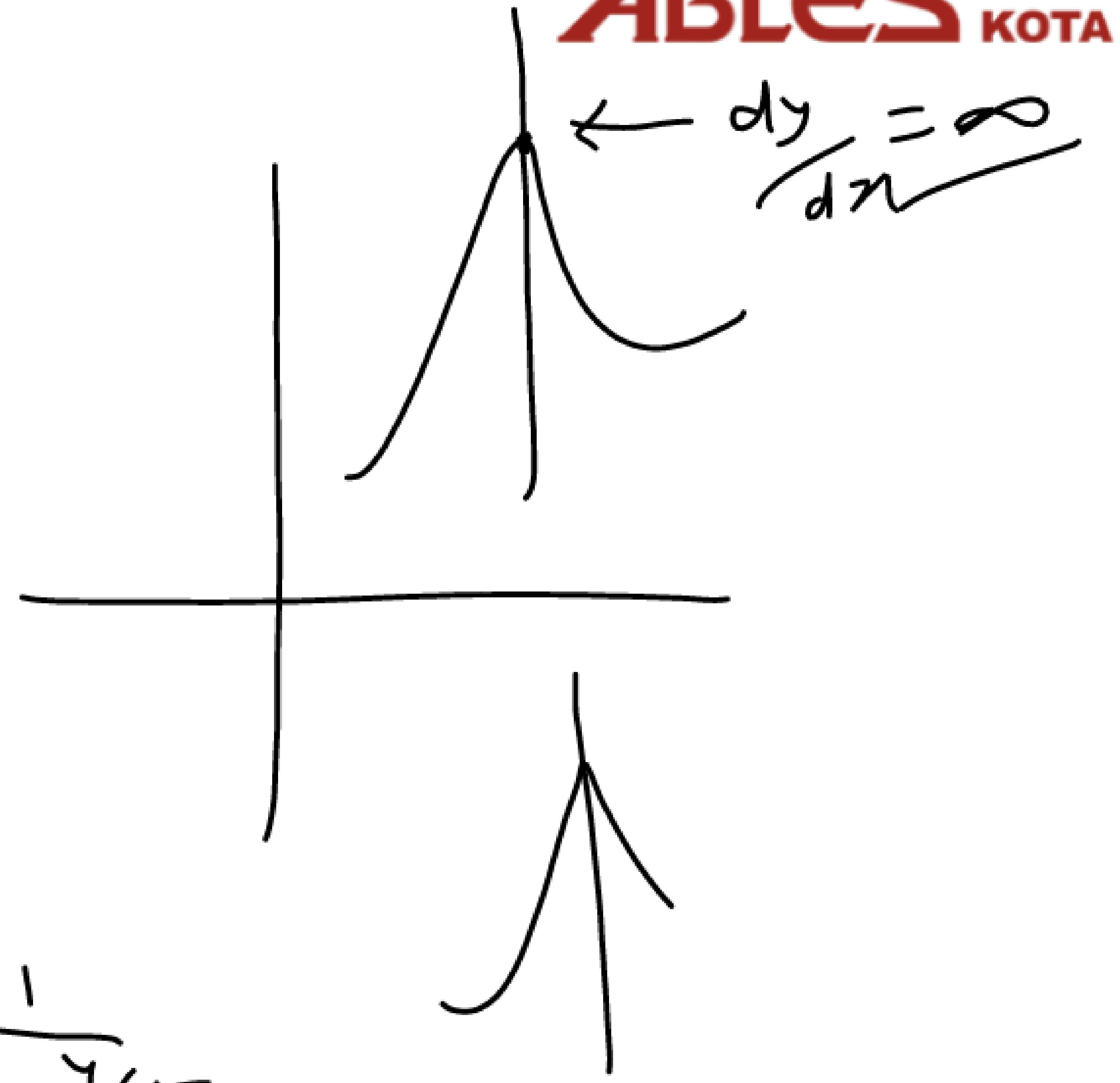
The curve $y = x^{\frac{1}{5}}$ at (0, 0) has

(a) a vertical tangent (parallel to y-axis)

(b) a horizontal tangent (parallel to x-axis)

$\boxed{\text{(c)}}$ no oblique tangent

(d) no tangent φ



Solⁿ:-

$$y = x^{1/5}$$

$$\frac{dy}{dx} = \frac{1}{5} x^{\frac{1}{5} - 1} = \frac{1}{5} x^{-4/5} = \frac{1}{5} \frac{1}{x^{4/5}}$$

$$\left.\frac{dy}{dx}\right)_{(0,0)} = \frac{1}{5} \times \frac{1}{(0)} = \infty \rightarrow \text{tangent}$$

The smallest value of the polynomial

$x^3 - 18x^2 + 96x$ in [0, 9] is

(a) 126 $\quad f'(x) = 3x^2 - 36x + 96$ $\quad$ (b) 0

(c) 135 $\quad \Rightarrow 3[x^2 - 12x + 32] = 0$ $\quad$ (d) 160

$$x^2 - 12x + 32 = 0$$

$$x^2 - 8x - 4x + 32$$

$$x(x-8) - 4(x-8)$$

$$x = 4, 8$$

$$f(x) = x^3 - 18x^2 + 96x$$

$$f(0) = 0$$

$$f(4) =$$

$$f(8) =$$

$$f(9) =$$

A point c in the domain of a function f is called a critical point of f if

I. $f'(c) = 0$

II. f is not differentiable at c.

Choose the correct option

(a) Either I or II are true

(b) Only I is true

(c) Only II is true

(d) Neither I nor II is true

The function $f(x) = x^2 \log x$ in the interval [1, e] has

(a) a point of maximum and minimum
(b) a point of maximum only
(c) no point of maximum and minimum in [1, e]
(d) no point of maximum and minimum

$f(x) = x^2 \log x$

$f'(x) = x^2 \cdot \frac{1}{x} + \log x \cdot 2x$

$f'(x) = x + 2x \log x$ — ①

Let $f'(x) = 0 = x(1 + 2\log x)$

So:- $\boxed{x = 0}$ ① or $\log x = -\frac{1}{2}$

$\boxed{x = e^{-1/2}}$ ②

Diff eq ① again

$f''(x) = 1 + 2\left[x \cdot \frac{1}{x} + \log x \cdot 1\right]$

$f''(x) = 1 + 2(1 + \log x)$

at $x = 1 \rightarrow f''(1) = 1 + 2(1) = 3 > 0 \rightarrow$ So:- $f(x)$ has min value at $\boxed{x = 1}$.

at:- $x = e \rightarrow f''(e) = 1 + 2[1 + 1] = 5 > 0 \rightarrow$ So $f(x)$ has min value at $x = e$

$$\text{If } y = \frac{ax-b}{(x-1)(x-4)}$$ has a turning point $P(2,-1)$, then the value of a and b respectively, are

(a) 1, 2 (b) 2, 1

(c) 0, 1 (d) 1, 0

x^2-5x+4

Here the curve y has a turning point P(2, −1) ∴ tangent $\frac{dy}{dx} = ?$

Sol- $\frac{dy}{dx} = \frac{(x^2-5x+4)a - (ax-b)(2x-5)}{(x^2-5x+4)^2}$

NOW $\left.\frac{dy}{dx}\right|_{(2,-1)} = 0 \Rightarrow \frac{(4-10+4)a-(2a-b)(4-5)}{(4-10+4)^2} = 0$

Here P(2, −1) lies on the curve y so:- satisfy the curve.

So!- $-1 = \frac{a(2)-b}{(2-1)(2-4)}$

$\Rightarrow -1 = \frac{2a-b}{-2} \Rightarrow \boxed{2a-b=2}$ — (1)

$\Rightarrow -2a-(2a-b)(-1) = 0$

$\Rightarrow -2a+(2a-b) = 0$

$\Rightarrow -2a+2 = 0$

$\Rightarrow a = -\frac{2}{-2} = 1 \Rightarrow a=1$

$\Rightarrow 2(1)-b = 2 \Rightarrow b = 0$

ABLES® KOTA

ABLES® KOTA

An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. The cost of the material will be least when depth of the tank is

(a) twice of its width

(b) half of the width

(c) equal to its width

(d) None of these

let $l = x$

$b = x$

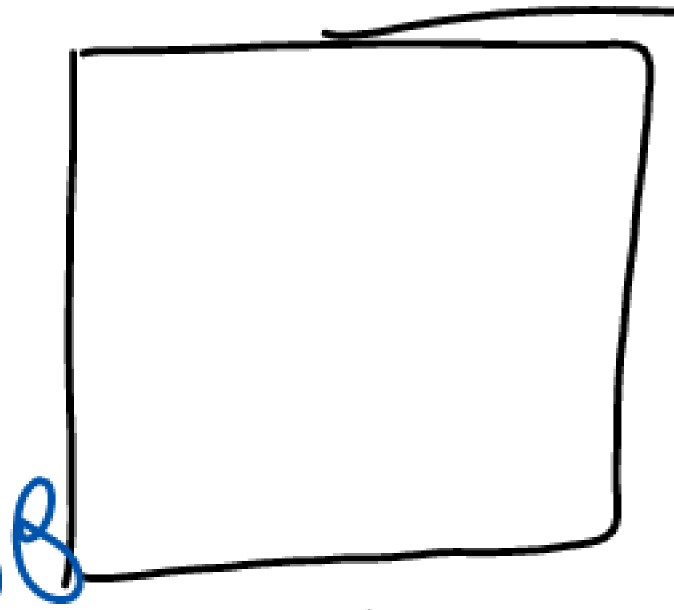
$h = y$

so $A = x^2 + 4x \cdot \frac{V}{x^2}$

diff

double diff

Surface Area



least → SA − top area

$A = 2(lb + bh + hl) - l \times b$

$= 2[x^2 + xy + xy] - x^2$

$A = 2x^2 + 4xy - x^2$

$f(x) \leftarrow A = x^2 + 4xy$

$y = \frac{x}{2}$

$x = 2y$

vol. $= l \times b \times h$

$V = x \times x \times y$

$V = x^2 y \rightarrow y = \frac{V}{x^2}$

Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a.

(a) 2a/3

(b) $\frac{2a}{\sqrt{3}}$

(c) a/3

(d) a/5

The maximum value of $\left(\frac{1}{x}\right)^{x}$ is

(a) e

(b) e^{e}

(c) $e^{\frac{1}{e}}$

(d) $\left(\frac{1}{e}\right)^{\frac{1}{e}}$

1	2	3	4	5	6	7	8	9	10
B	C	B	B	A	C	D	B	B	C