AOD

Ques:- The minimum value of $[px + qy]$ when $[xy = r^2]$ $(x^{-2})' = -2x^{-3} = \frac{-2}{x^3}$

Sol^n:- let $A = px + qy$ & $xy = r^2 \Rightarrow \left[y = \frac{r^2}{x}\right]$ — (1)

So:- $\left[A = px + q \cdot \frac{r^2}{x}\right]$ → find → min. value of A.

→ Diff → A wrt x :- $\frac{dA}{dx} = p + q \cdot r^2\left(\frac{-1}{x^2}\right)$ — (2)

let $\frac{dA}{dx} = 0 \Rightarrow p - \frac{q \cdot r^2}{x^2} = 0 \Rightarrow p = \frac{q \cdot r^2}{x^2} \Rightarrow x^2 = \left(\frac{q}{p}\right) \cdot r^2 \Rightarrow \boxed{x = r\sqrt{\frac{q}{p}}}$ (3)

put x in eq (1) :- $y = r^2 \cdot \frac{1}{r \cdot \sqrt{q/p}} = \boxed{r\sqrt{\frac{p}{q}} = y}$ (4)

So again diff. eq. (2) :- $\frac{d^2A}{dx^2} = 0 - q \cdot r^2\left(\frac{-2}{x^3}\right) = \frac{2 q r^2}{x^3} > 0$ So:- A has min. value & $x = r\sqrt{q/p}$ & $y = r\sqrt{p/q}$.

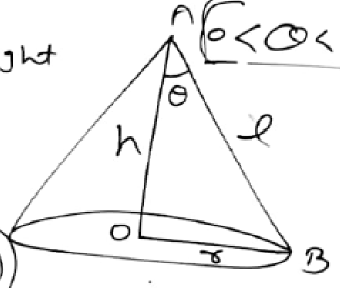
So:- $A = p \cdot r\sqrt{\frac{q}{p}} + q \cdot r\sqrt{p/q} = r(\sqrt{pq}) + r\sqrt{pq} = \boxed{2r\sqrt{pq}}$ Ans

AOD

Ques:- Show that the semi-vertical angle of the cone of the maximum vol & of given slant height is $\tan^{-1}\sqrt{2}$ proved

Soln:- let h, r & l are height, Radius & slant height of the cone & θ will be semi-vertical angle.

$0<\theta<90^\circ$



let in ΔAOB:- $\sin\theta = \frac{r}{l} \Rightarrow [r = l\sin\theta]$

& $\cos\theta = \frac{h}{l} \Rightarrow [h = l\cos\theta]$

So: volume of cone $\rightarrow$ $[V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (l^2\sin^2\theta)(l\cos\theta)]$

So: Diff V wrt θ: $\frac{dv}{d\theta} = \frac{1}{3}\pi l^3[\sin^2\theta\cdot(-\sin\theta) + \cos\theta\cdot(2\sin\theta\cos\theta)]$

$\Rightarrow \frac{dv}{d\theta} = \frac{1}{3}\pi l^3[-\sin^3\theta + 2\sin\theta\cdot\cos^2\theta] \Rightarrow$ let $\frac{dv}{d\theta} = 0 \Rightarrow -\sin^3\theta + 2\sin\theta\cdot\cos^2\theta = 0$

$\Rightarrow \sin^2\theta = 2\cos^2\theta$

$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = 2 \Rightarrow \tan^2\theta = 2 \Rightarrow \tan\theta = \sqrt{2}$ $\quad -\sqrt{2}$

$\theta = \tan^{-1}\sqrt{2}$

Diff again: $\frac{dv}{d\theta} = \frac{1}{3}\pi l^3[-3\sin^2\theta\cdot\cos\theta + 2\{\sin\theta\cdot 2\cos\theta\cdot(-\sin\theta) + \cos^2\theta\cdot\cos\theta\}]$

$= \frac{1}{3}\pi l^3[-3\sin^2\theta\cdot\cos\theta - 4\sin^2\theta\cdot\cos\theta + 2\cos^3\theta]$

$= \frac{1}{3}\pi l^3\{-3\tan^2\theta - 4\tan^2\theta + 2\} = \frac{1}{3}\pi l^3[-7\tan^2\theta + 2] < 0$

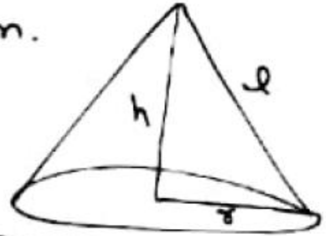
So: Vol is max. at $\theta = \tan^{-1}\sqrt{2}$ H.P.

HAOD

Ques. Show that the Right circular cone of least curved surface & $\boxed{\text{given vol.}}$ has an altitude equal to $\sqrt{2}$ time the radius of the base.

Sol^n. prove : $\boxed{h = \sqrt{2}\,r}$ & curved surface area is least → function.

& volume is given. $\Rightarrow V = \frac{1}{3}\pi r^2 h \Rightarrow \boxed{h = \frac{3V}{\pi r^2}}$ —— (1)



∵ Curve surface area $\Rightarrow S = \underline{\pi r l} = \pi r \sqrt{h^2 + r^2}$

$$\Rightarrow \boxed{S = \pi r \sqrt{\frac{9V^2}{\pi^2 r^4} + \frac{r^2}{1}}} = \pi r \sqrt{\frac{9V^2 + \pi^2 r^6}{\pi^2 r^4}} = \frac{\cancel{\pi r}}{\cancel{\pi} r^{\cancel{2}}} \sqrt{9V^2 + \pi^2 r^6}$$

$$\Rightarrow \boxed{S = \frac{1}{r}\sqrt{9V^2 + \pi^2 r^6}}$$

u.v

$$\rightarrow \frac{dS}{dr} = \underline{\qquad\qquad} = 0 \Rightarrow \boxed{\underline{r}^6 = ?} \rightarrow V$$

$$\boxed{r^6 = \frac{9V^2}{2\pi^2}}$$

put value of

$$\boxed{h = \sqrt{2}\,r}$$

$\frac{d^2S}{dr^2}$ at r^6 > 0 → So Surface area is least.

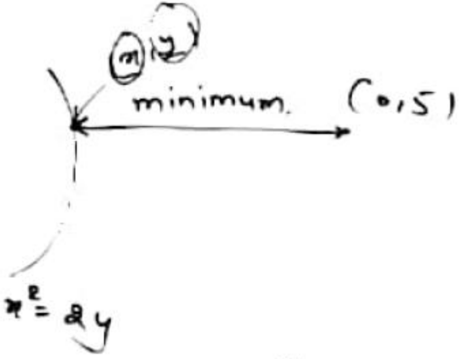
HAOD

Qus: The point on the curve $x^2 = 2y$ which is nearest to the point (0,5) is

(A) $(2\sqrt{2}, 4)$ (B) $(2\sqrt{2}, 0)$ (C) $(0, 0)$ (D) $(2, 2)$

Sol:- let a point P(x,y) on the curve $x^2 = 2y$ which is nearest to the point (0,5).

So:- $P(x,y) = P\left(x, \frac{x^2}{2}\right)$ & $Q(0,5)$



So: we have to find distance b/w P & Q.

So:- $PQ = \sqrt{(x-0)^2 + \left(\frac{x^2}{2} - 5\right)^2}$

$\Rightarrow (PQ)^2 = x^2 + \frac{x^4}{4} + 25 - 2 \times \frac{x^2}{2} \times 5$

$\Rightarrow \left[(PQ)^2 = x^2 + \frac{x^4}{4} + 25 - 5x^2\right] = A = f(x)$

Here we to show that the Distance A is min

So Diff. A → $\frac{dA}{dx} = 2x + \frac{4x^3}{4} - 10x$

Let $\frac{dA}{dx} = 0 \Rightarrow -8x + x^3 = 0$

$\Rightarrow 8x = x^3$

$\Rightarrow x^2 = 8$

$\therefore \boxed{x = \sqrt{8} = \pm 2\sqrt{2}}$

Diff again → $\frac{d^2A}{dx^2} = 2 + 3x^2 - 10 = \boxed{-8 + 3x^2} > 0$ ∴ A or Distance is min. when $x = \pm 2\sqrt{2}$

So: $y = \frac{8}{2} = 4$ So Point $P(2\sqrt{2}, 4)$ Ans