$$\frac{dv}{dr} = \frac{2}{3}\pi r R + \frac{1}{3}\pi\left[\frac{-r^3 + (R^2 - r^2)(2r)}{\sqrt{R^2 - r^2}}\right]$$

diff $$\frac{d^2v}{dr^2} = \frac{2}{3}\pi R + \frac{1}{3}\pi\left[\frac{\sqrt{R^2-r^2}\left\{-3r^2 + (R^2-r^2)2 + 2r(-2r)\right\}}{(R^2-r^2)}\right]$$

Here when we put $r^2 = \frac{8}{9}R^2$ $\frac{d^2v}{dr^2}$ will become $-$ive

$$-r^3 + (R^2 - r^2)(2r) \frac{1}{\cancel{2}\sqrt{R^2-r^2}} \times -\cancel{2}r$$

$$\boxed{r^2 = \frac{8}{9}R^2}$$

$$\frac{d^2v}{dr^2} = \frac{2}{3}\pi R + \frac{1}{3}\pi \frac{1}{(R^2-r^2)}\left[\frac{(R^2-r^2)\left[-3r^2 + 2R^2 - 2r^2 - 4r^2 - r^3 + (2R^2r - 2r^3)r\right]}{\sqrt{R^2-r^2}}\right]$$

< 0

$\Rightarrow \quad 4R^2(R^2 - r^2) = (3r^2 - 2R^2)^2$

$\Rightarrow \quad \cancel{4R^4} - 4R^2r^2 = 9r^4 + \cancel{4R^4} - 2 \times 3r^2 \times 2R^2$

$\Rightarrow \quad -4R^2r^2 = 9r^4 - 12R^2r^2$

$\Rightarrow \quad 8R^2\cancel{r^2} = 9\cancel{r^4} \Rightarrow 8R^2 = 9r^2 \Rightarrow \boxed{r^2 = \frac{8}{9}R^2}$

put r^2 in eq. (2) :- $h = R + \sqrt{R^2 - r^2}$

$h = R + \sqrt{R^2 - \frac{8R^2}{9}} = R + \sqrt{\frac{R^2}{9}} = R + \frac{R}{3} = \boxed{\frac{4R}{3} = h}$

So Vol. of Cone $= \frac{1}{3}\pi \frac{8}{9}R^2 \times \frac{4R}{3} = \frac{8}{27} \times \frac{4}{3}\pi R^3$

H.P.

$\boxed{\text{Vol. of Cone} = \frac{8}{27} \times \text{Vol. of Sphere.}}$

So vol. of Cone $= V = \frac{1}{3}\pi r^2 \left[R + \sqrt{R^2 - r^2}\right]$

$$\boxed{V = \frac{1}{3}\pi r^2 R + \frac{1}{3}\pi r^2 \sqrt{R^2 - r^2}}$$

$\Rightarrow$ Diff $\rightarrow$ $\frac{dv}{dr} = \frac{2}{3}\pi r R + \frac{1}{3}\pi \left(r^2 \cdot \frac{1}{\cancel{2}\sqrt{R^2 - r^2}} \times (-\cancel{2}r) + \frac{\sqrt{R^2 - r^2}\,(2r)}{1} \right)$

$$\boxed{\frac{dv}{dr} = \frac{2}{3}\pi r R + \frac{1}{3}\pi \left[\frac{-r^3 + (R^2 - r^2)(2r)}{\sqrt{R^2 - r^2}} \right]}$$

$\rightarrow$ let $\boxed{\frac{dv}{dr} = 0}$

$\Rightarrow \frac{2}{3}\pi r R + \frac{1}{3}\pi \left[\frac{-r^3 + (R^2 - r^2)(2r)}{\sqrt{R^2 - r^2}} \right] = 0$

$\Rightarrow \frac{2}{\cancel{3}}\cancel{\pi}\cancel{r}R = \frac{1}{\cancel{3}}\cancel{\pi} \left[\frac{r^{\cancel{3}\,2} - (R^2 - r^2)(2\cancel{r})}{\sqrt{R^2 - r^2}} \right]$

$\Rightarrow 2R = \frac{r^2 - 2R^2 + 2r^2}{\sqrt{R^2 - r^2}} \Rightarrow 2R\sqrt{R^2 - r^2} = 3r^2 - 2R^2$

$\Rightarrow 4R^2 (R^2 - r^2) = (3r^2 - 2R^2)^2$

square it

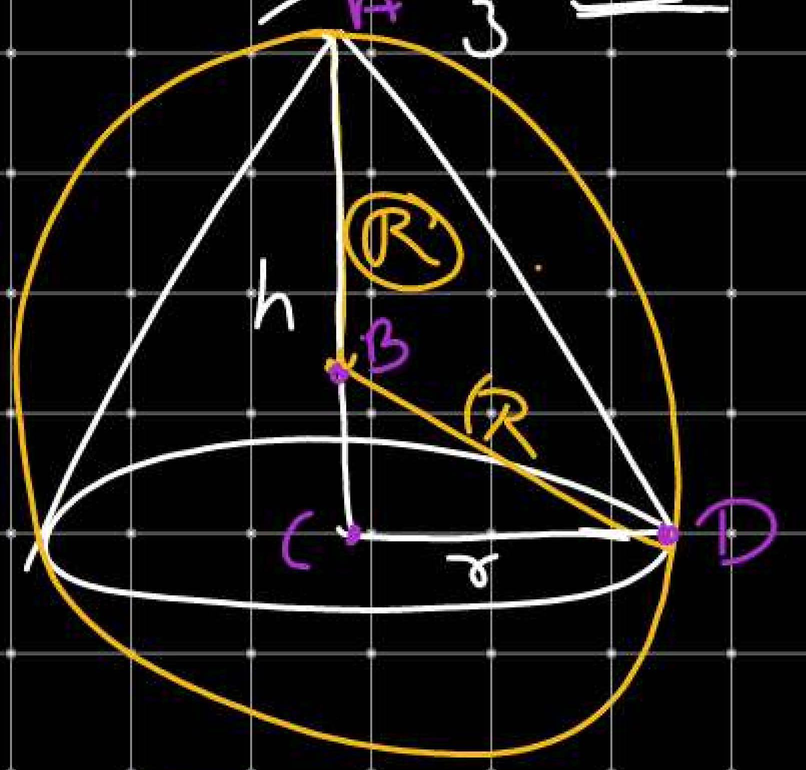
Q. Prove that the vol of the largest cone that can be inscribed in a sphere of Radius R is $\frac{8}{27}$ of the vol. of the sphere.

Sol^n: given

vol. of cone is largest

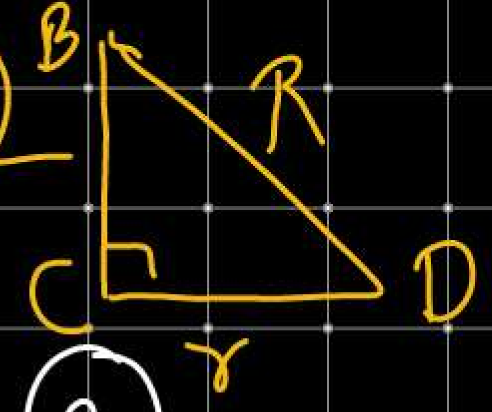
prove:- vol. of cone $= \frac{8}{27} \times$ vol. of sphere

$\rightarrow \frac{4}{3}\pi R^3$



→ let Radius of sphere $= R$
& radius & height of the cone is r & h.

Sol:- Volume of cone $= f(r) = \frac{1}{3}\pi r^2 h$ — (1)



$\sqrt{R^2-r^2}$

NOW :- in Right ΔACD :- $h = AC = AB + BC$

$h = R + BC \Rightarrow \boxed{h = R + \sqrt{R^2 - r^2}}$ — (2)

#AOD#

Ques:- A wire of length 28 m is to be cut into 2 pieces is to be made into a square & the other into a circle. What should be the length of the 2 pieces so that the combined area of the sq. & circle is minimum.

$l \quad | \quad (28-l)$

① 28 m ②

↓ Square ↓ circle

$$\boxed{l = \frac{112}{\pi+4}}$$

sq

$4 \times x = l$

side $\boxed{x = l/4}$

$A = \text{side}^2 + \pi r^2$

$$\boxed{A = \frac{l^2}{4^2} + \pi\frac{(28-l)^2}{(2\pi)^2}}$$

$f(l)$

$2\pi r = (28-l)$

$$\boxed{r = \frac{28-l}{2\pi}}$$

ABLES KOTA

$$\frac{dv}{dr} = \frac{2}{3}\pi r R + \frac{1}{3}\pi\left[\frac{-r^3 + (R^2 - r^2)(2r)}{\sqrt{R^2 - r^2}}\right]$$

diff

$$\frac{d^2v}{dr^2} = \frac{2}{3}\pi R + \frac{1}{3}\pi\left[\frac{\sqrt{R^2-r^2}\left\{-3r^2 + (R^2 - r^2)2 + 2r(-2r)\right\}}{(R^2 - r^2)}\right.$$

$$- \left. \frac{-r^3 + (R^2 - r^2)(2r)}{} \frac{1}{\cancel{2}\sqrt{R^2 - r^2}} \times -\cancel{2}r\right]$$

Here when we put $r^2 = \frac{8}{9}R^2 \to \frac{d^2v}{dr^2}$ will become -ive

$$\boxed{r^2 = \frac{8}{9}R^2}$$

$$\frac{d^2v}{dr^2} = \frac{2}{3}\pi R + \frac{1}{3}\pi \frac{1}{(R^2 - r^2)}\left[\frac{(R^2 - r^2)\left[-3r^2 + 2R^2 - 2r^2 - 4r^2 - r^3 + (2R^2 r - 2r^3)r\right.}{\sqrt{R^2 - r^2}}\right]$$

$$< 0$$