AOD

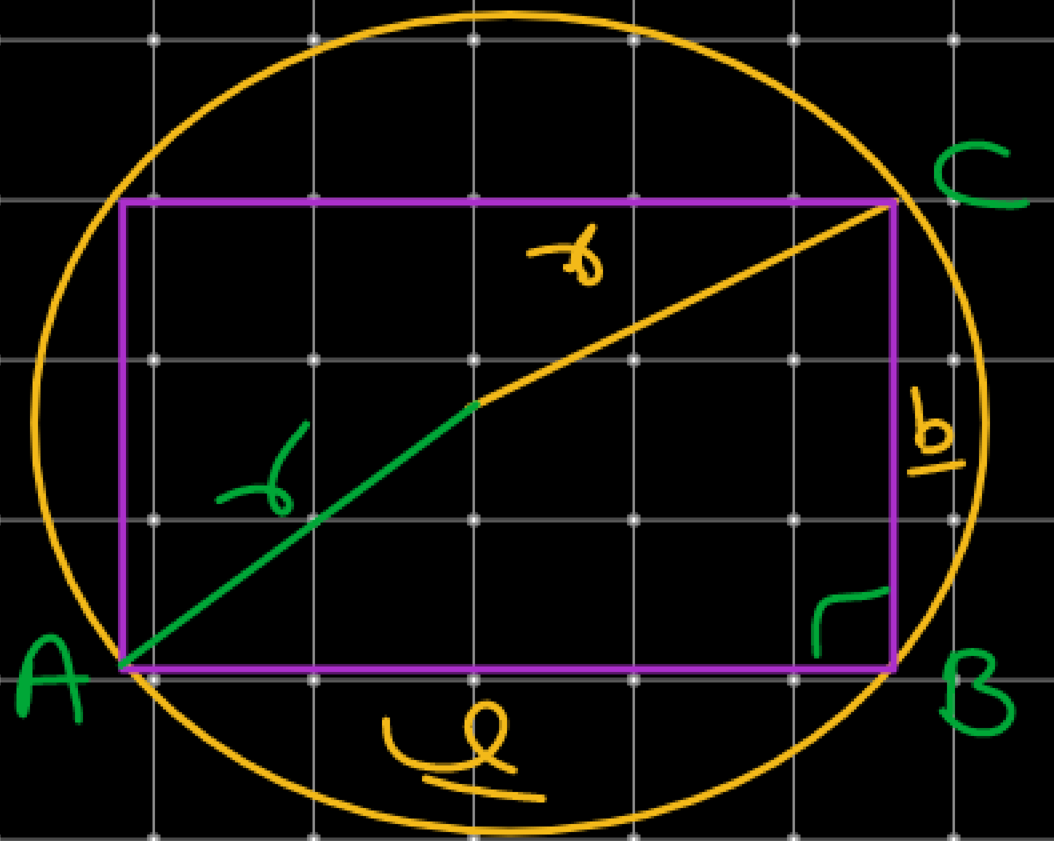
Ques 1 - Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

Sol^n: let

length of Rect. $= l$

breath of Rect $= b$

& Radius of circle $= r$



from fig.

in $\Delta ABC \Rightarrow (2r)^2 = l^2 + b^2$

$\Rightarrow 4r^2 = l^2 + b^2 \Rightarrow b^2 = 4r^2 - l^2$

$\Rightarrow \boxed{b = \sqrt{4r^2 - l^2}}$ —— A

Rec $= l \times b = A$

Sol Area of Rect $\Rightarrow \boxed{A = l \times b = l \times \sqrt{4r^2 - l^2} = f(l)}$

Now diff.

$\Rightarrow f(l) = l \times \sqrt{4r^2 - l^2}$

$\Rightarrow f'(l) = l \times \frac{1}{\cancel{2}\sqrt{4r^2-l^2}} \times -\cancel{2}l + \sqrt{4r^2-l^2} \times 1$

$\boxed{\frac{2r^2 \times (-4\sqrt{2}r)}{(2r^2)^{3/2}} < 0}$

$\Rightarrow f'(l) = \frac{-l^2 + (4r^2 - l^2)}{\sqrt{4r^2-l^2}} = \boxed{\frac{4r^2 - 2l^2}{\sqrt{4r^2-l^2}} = f'(l)} \longrightarrow (1)$

Let $f'(l) = 0$

$\Rightarrow 4r^2 - 2l^2 = 0 \Rightarrow \cancel{2}l^2 = \cancel{4}^{2}r^2 \Rightarrow \boxed{l = \sqrt{2}r} \longrightarrow (11)$

$\rightarrow$ put $l = \sqrt{2}r$ in eq. (A) :- $b = \sqrt{4r^2 - 2r^2} = \sqrt{2r^2} \Rightarrow \boxed{b = \sqrt{2}r}$ (3)

From eq. (2) & (3) :- it is clear that Rect is sq.

Diff again (1) :- $f''(l) = \frac{\sqrt{4r^2-l^2} \times (-4l) - (4r^2 - 2l^2) \times \frac{1}{\cancel{2}\sqrt{4r^2-l^2}} \times -\cancel{2}l}{(4r^2 - l^2)}$

$\Rightarrow f''(l) = \frac{(4r^2 - l^2)(-4l) + l(4r^2 - 2l^2)}{(4r^2 - l^2)\sqrt{4r^2-l^2}} = \frac{(4r^2 - 2r^2)(-4\sqrt{2}r) + \sqrt{2}r(0)}{(4r^2 - 2r^2)^{3/2}}$

So Here it is clear that $f''(l)$ at $\boxed{l = \sqrt{2}r}$

is < 0. i.e. the area of Rect is max.

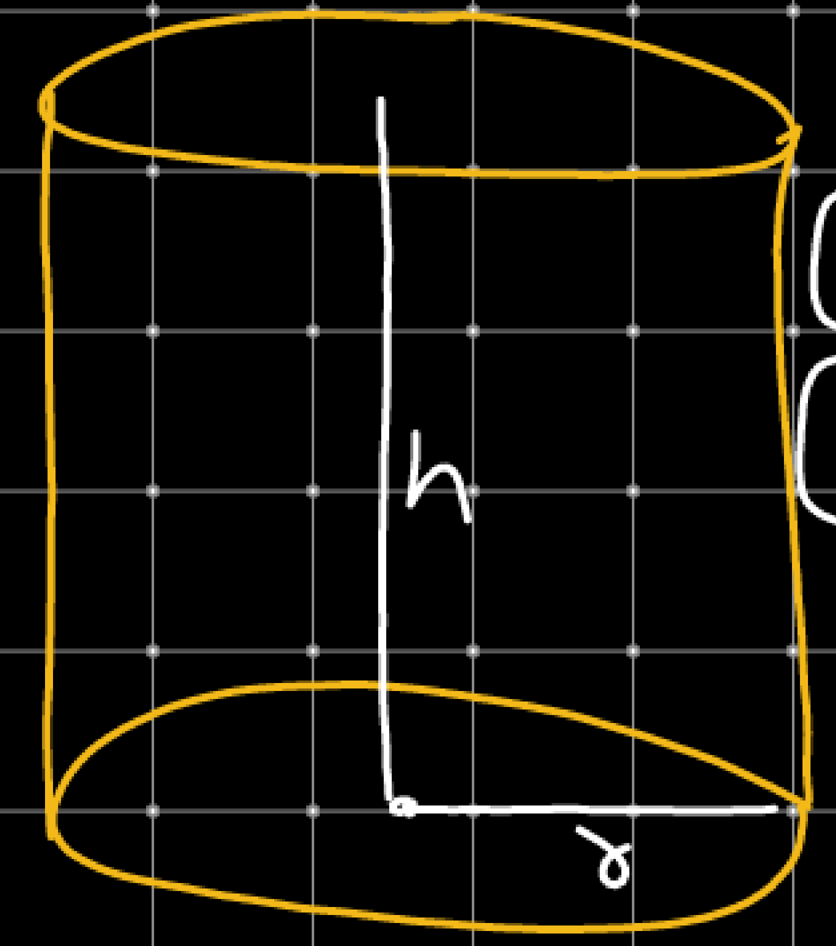
& $l = b = \sqrt{2}r$ therefor the Rect is a sq.

Done

Q. Show that the right circular cylinder of given Surface & $\boxed{\text{max. volume}}$ is such that its Height is equal to the Diameter of the base.

Soln :- Let radius of cylinder $= r$

& Height of cylinder $= h$



given:- $SA \Rightarrow \boxed{A = 2\pi r(h+r) = 2\pi rh + 2\pi r^2}$

$\Rightarrow A - 2\pi r^2 = 2\pi rh \Rightarrow h = \frac{A}{2\pi r} - \frac{2\pi r^2}{2\pi r}$

$\Rightarrow \boxed{h = \frac{A}{2\pi}\left(\frac{1}{r}\right) - r}$ — (1)

∴ we have to find: max. volume $\Rightarrow \boxed{V = \pi r^2 h}$

So! volume $\Rightarrow V = \pi r^2\left[\frac{A}{2\pi}\left(\frac{1}{r}\right) - r\right] \Rightarrow \boxed{V = \frac{A}{2}r - \pi r^3} = f(r)$

$\Rightarrow f(r) = \frac{A}{2} r - \pi r^3 \longrightarrow \text{Diff} \rightarrow \boxed{f'(r) = \frac{A}{2} - 3\pi r^2}$

let $f'(r) = 0 \Rightarrow \frac{A}{2} - 3\pi r^2 = 0 \Rightarrow 3\pi r^2 = \frac{A}{2} \Rightarrow \boxed{r^2 = \frac{A}{6\pi}}$ — (2)

from eq. (1) & (2) :-

$h \Rightarrow \frac{A}{2\pi}\left(\frac{1}{r}\right) - r \Rightarrow h = \frac{\cancel{6}^{3}\pi r^{\cancel{2}}}{\cancel{2}\pi \cdot \cancel{r}} - r \Rightarrow \boxed{h = 2r}$

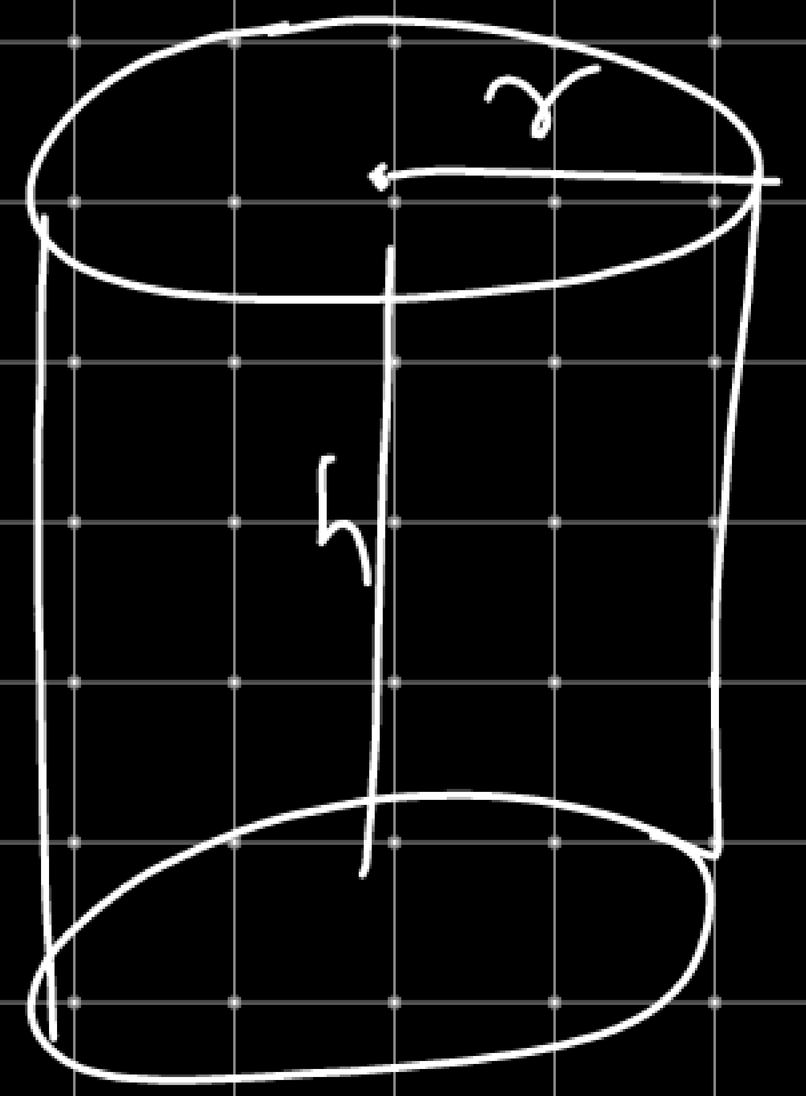
Diff again $f'(r)$ wrt $r \rightarrow f''(r) = 0 - 6\pi r < 0$

Hence it is clear that for a given surface area volume of cylinder is max. when $\boxed{h = 2r}$

Ques:- H.W.

Of all the closed cylindrical can of a given volu. of 100 cm^3. find the dimension of the can which has the minimum surface area.

r & h



Volume → $V = 100$

$$\pi r^2 h = 100$$

$$\boxed{h = \frac{100}{\pi r^2}}$$

find:- $f(x) = 2\pi r h + 2\pi r^2$

$f'(x) = 0$

$f''(x) = > 0$