Ex:- Find two no. x & y such that $x+y=60$ & xy^3 is maximum

Sol^n:- 2 nu. are → x & y → $x+y=60 \Rightarrow \boxed{y=60-x}$

∵ given :- xy^3 is max. ⇒ let $f(x)=xy^3$

∴ $\boxed{f(x)=x\cdot(60-x)^3}$

Diff → $f'(x) = -x\cdot 3(60-x)^2 + (60-x)^3$

$= (60-x)^2[-3x+60-x]$

$f'(x) = (60-x)^2[60-4x]$

let $f'(x)=0 \Rightarrow (60-x)^2(60-4x)=0$

then $x=60$ $x=15$ ✓

Not possible

Now Diff $f'(x)$ again

$f''(x) = (60-x)^2(-4) + (60-4x)\,2(60-x)(-1)$

$= (60-x)\,2[-2(60-x)+(60-4x)(-1)]$

$= 2(60-x)[-120+2x-60+4x]$

$= 2(60-x)[6x-180]$

at $x=15 \to f''(x) \to -ive$

i.e. $f(x)=xy^3 \to$ maximum

So $x=15$ & $y=45$

H.W. Find 2 +ive no. x & y such that their sum is 35 & product $x^2 \cdot y^5$ is maximum.

Q. Find 2 +ive no. whose sum 16 & the sum of of whose cubes is minimum.

ABLES KOTA

Sol^n:- given let $x + y = 16 \longrightarrow \boxed{y = (16 - x)}$

find $\Rightarrow \boxed{x^3 + y^3 \longrightarrow \text{minimum}}$

let $f(x) = x^3 + y^3$

$f(x) = x^3 + (16 - x)^3$

Diff:- $f'(x) = 3x^2 + 3(16-x)^2 \times (-1)$

$= 3x^2 - 3(16-x)^2$

let $f'(x) = 0 \Rightarrow 3x^2 - 3(16-x)^2 = 0$

$\Rightarrow 3[x^2 - (16-x)^2] = 0$

$\Rightarrow 3[\{x - (16-x)\}\{x + 16 - x\}] = 0$

$3[(2x-16)(16)] = 0$

$2x - 16 = 0 \Rightarrow \boxed{x = 8}$

Diff. $f'(x)$ again:-

$f''(x) = 6x - 6(16-x)(-1)$

$= 6[x + 96 - 6x]$

$= 6[96 - 5x]$

put $x = 8 \rightarrow f''(x) \rightarrow$ +ive

$\therefore f(x) = x^3 + y^3$ is minimum

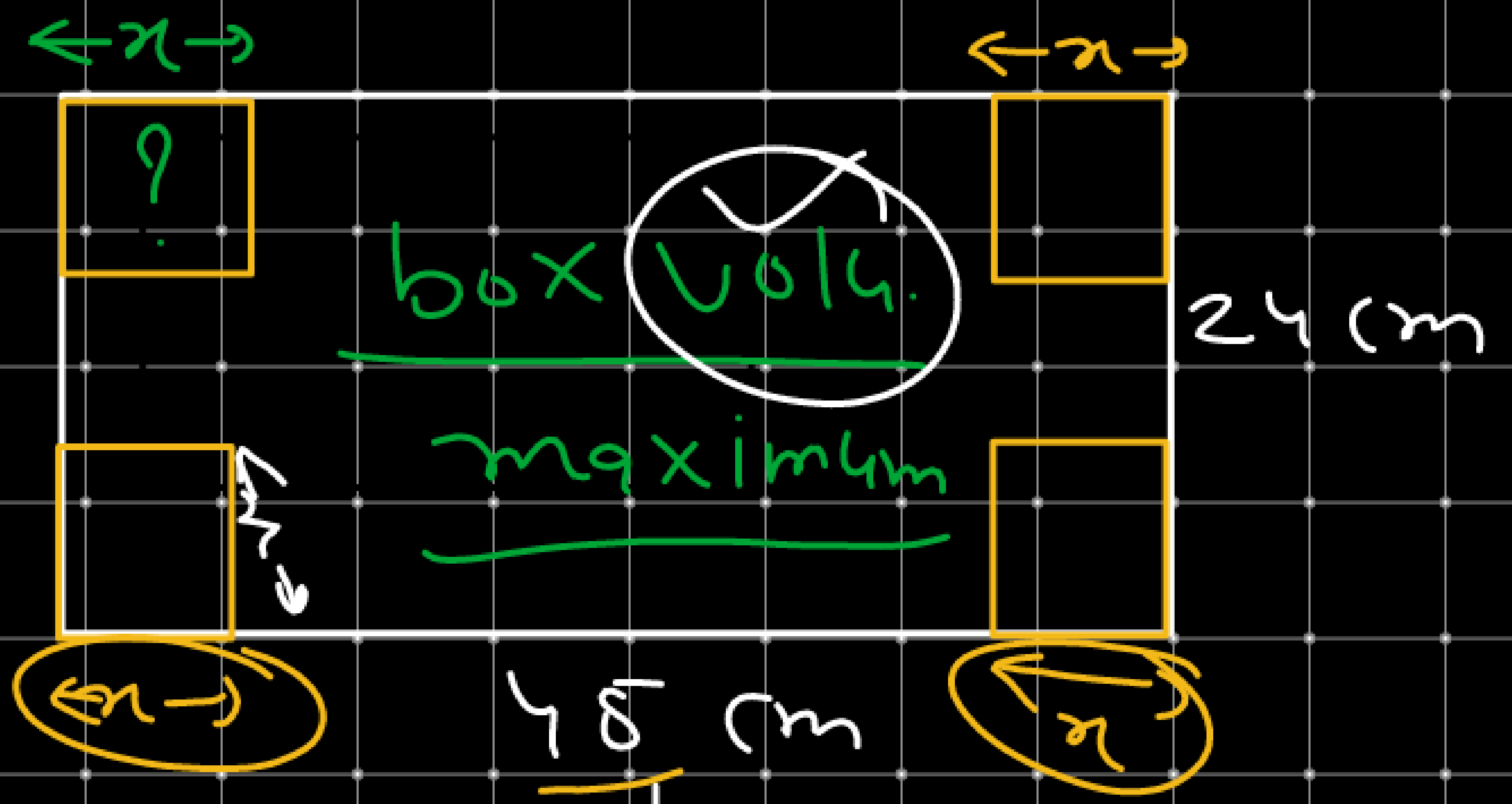
at $\boxed{x = 8}$ & $\boxed{y = 8}$

Q. A Rect. sheet 45 cm × 24 cm is to be made into a box without top, by cutting off square from each corner & folding up the flaps. What should be the side of sq. to be cut off so that the volume of the box is the maximum possible?

Soln:- Rect. sheet length = 45 cm

Breath = 24 cm

Let the sq. to be cut has side = x cm



Now Box length $= (45-2x)$ cm

Breath $= (24-2x)$ cm

height $= x$ cm

box → cuboid

So volume of box $= l \times b \times h$

$= (45-2x)(24-2x)x$

Let $f(x)$ = Volume $= (45-2x)(24x-2x^2)$ → max.

ABLES KOTA

$f(x) = (45-2x)(24x-2x^2)$

$\rightarrow f'(x) = (45-2x)(24-4x) + (24x-2x^2)(-2)$

Diff $\rightarrow$ $f''(x) = (45-2x)(-4) + (24-4x)(-2) - 2(24-4x)$

$f''(x) = [-4(45-2x) - 4(24-4x)]$

Now $f'(x) = 0 \Rightarrow (45-2x)(24-4x) - 2(24x-2x^2) = 0$

$\Rightarrow 45 \times 24 - 180x - 48x + 8x^2 - 48x + 4x^2 = 0$

$\Rightarrow 12x^2 - 276x + 1080 = 0$

$\Rightarrow x^2 - 23x + 90 = 0$

$\Rightarrow x^2 - 18x - 5x + 90 = 0$

$\Rightarrow x(x-18) - 5(x-18) = 0$

$\Rightarrow x = 18 \quad x = 5$

N.P.

So: $x = 5$ will be posible.

& at $x = 5 \rightarrow f''(x) = -140 - 16 =$

$= -156 < 0$

Hence it is clear that at $x = 5$ volume will be max.

So side of squ. to be cut is 5 cm up

Q. a sq. piece of 18cm side is to be made into a box without top, by cutting a square from each corner & folding up the flaps to form the box. What should be the side of the sq. to be cut off so that the volume of box is maximum.