Ex:- Find absolute max. and min.

i) $f(x) = \sin x + \cos x \; ; \; x \in [0, \pi]$

Sol^n:- $f'(x) = \cos x - \sin x$

let $f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

$\Rightarrow \sin x = \cos x$

$\Rightarrow \cancel{\tan} x = 1 = \cancel{\tan} \pi/4$

$\boxed{x = \frac{\pi}{4}}$

Sol:- $f(x) \rightarrow x = 0 \Rightarrow f(0) = \sin 0 + \cos 0 = 1$

$\rightarrow x = \pi/4 \Rightarrow f(\pi/4) = \sin \pi/4 + \cos \pi/4 = \sqrt{2}$

$\rightarrow x = \pi \rightarrow f(\pi) = \sin \pi + \cos(\pi) = -1$

So $f(x)$ has max. value at $x = \pi/4$ &

max. value $= \sqrt{2}$

$\rightarrow f(x)$ has min. value at $x = \pi$ &

min value $= -1$ Ans

iii) $f(x) = (x-1)^2 + 3 \quad ; x \in [-3, 1]$

Solⁿ :- $f'(x) = 2(x-1) \rightarrow$ let $f'(x) = 0 \Rightarrow 2(x-1) = 0$

$\Rightarrow \boxed{x = 1}$

So! at $\boxed{x = -3} \rightarrow f(-3) = (-3-1)^2 + 3 = 19$

$\boxed{x = 1} \Rightarrow f(1) = (1-1)^2 + 3 = 3$

So max. value occur at $x = -3$ & max. value $= 19$

min value —— " " $\boxed{x = 1}$ & min value $= 3$ Ans

Extra :- $f''(x) = 2(1) = 2 \rightarrow$ +ive $> 0 \rightarrow$ minima

So $\boxed{x = 1} \rightarrow$ minima

H.W

1 $f(x) = 4x - \frac{1}{2}x^2 \quad x \in \left[-2, \frac{9}{2}\right]$

$f(x) = x^3 \ ; \quad x \in [-2, 2]$

Q. find max. & min value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on interval $[0, 3]$

$\frac{27}{\frac{8}{216}} 5$

Sol^n:

$$f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$$

$$\rightarrow \therefore \left[f'(x) = 12x^3 - 24x^2 + 24x - 48\right]$$

$36x^2 - 48x + 24$

$36 \times 4 - 96 + 24$

$\downarrow$

+ive

let $f'(x) = 0 \Rightarrow 12x^3 - 24x^2 + 24x - 48 = 0 \Rightarrow 12\left[x^3 - 2x^2 + 2x - 4\right] = 0$

$\Rightarrow 12\left[x^2(x-2) + 2(x-2)\right] \Rightarrow 12(x-2)(x^2+2) = 0$

So! $x = 2$ & $x \in [0, 3]$

$\Rightarrow f(0) = 25$

$f(2) = \cancel{48} - 64 + \cancel{48} - \cancel{96} + 25 = -39$

$f(3) = 243 - 216 + 108 - 144 + 25 = 16$

max. value $\Rightarrow x = 0 \rightarrow y = 25$

min value $\Rightarrow x = 2 \rightarrow y = -39$

Q. at what points in $[0, 2\pi]$ does the fun. $\boxed{\sin 2x}$ attains its max value.

$\frac{\pi}{4}, \frac{5\pi}{4}$

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Solⁿ: $f(x) = \sin 2x \rightarrow$ diff $\rightarrow f'(x) = \cos 2x \cdot 2$

$\rightarrow$ let $f'(x) = 0 \Rightarrow 2\cos 2x = 0 \Rightarrow \boxed{\cos 2x = 0}$

$\Rightarrow 2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

So! at $x = 0 \Rightarrow f(0) = \sin 0 = 0$

$x = \frac{\pi}{4} \Rightarrow f(\pi/4) = \sin \pi/2 = 1$

$x = \frac{3\pi}{4} \rightarrow f\left(\frac{3\pi}{4}\right) = \sin 3\pi/2 = -1$

$x = \frac{5\pi}{4} \rightarrow f\left(\frac{5\pi}{4}\right) = \sin\left(\frac{5\pi}{2}\right) = 1$

$x = \frac{7\pi}{4} \rightarrow f\left(\frac{7\pi}{4}\right) = \sin\left(\frac{7\pi}{2}\right) = -1$

$x = 2\pi \rightarrow f(2\pi) = \sin 4\pi = 0$

Q. it is given that at $x=1$, $f(x) = x^4 - 62x^2 + ax + 9$ attains its max. value, on interval $[0,2]$, find value of a.

Sol^n:

$\because f(x) = x^4 - 62x^2 + ax + 9$

$\therefore \underline{f'(x)} = 4x^3 - 124x + a$

let $f'(x) = 0 \Rightarrow \boxed{4x^3 - 124x + a = 0}$

but it is given that at $x=1$ fun. has max value:

So! at $\boxed{x=1} \rightarrow \underline{4(1)^3 - 124(1)} + a = 0$

$\Rightarrow a = 120$ Ans

Q. find max & min value of $\boxed{x+\sin 2x}$ on $[0, 2\pi]$

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$$\boxed{x = \pi/3,\ 2\pi/3,\ 4\pi/3,\ 5\pi/3}$$

Q. Find two numbers whose sum is 24 & whose product is as large as possible.

12, 12 → 144

11, 13 → 143

10, 14 → 140

Solⁿ:- let I^{st} number → x

II^{nd} number ⇒ $\boxed{24-x}$

So product: $P(x) = x(24-x) = 24x - x^2 = P(x)$

So diff → $P'(x) = 24 - 2x$ → let → $P'(x) = 0 \Rightarrow 24 - 2x = 0$

$\boxed{x = 12}$

Now again diff → $P''(x) = -2 < 0$ → maxima product

So I^{st} no $= 12$

II^{nd} no $= 24 - 12 = 12$