A point on the parabola $y^2=18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$

(c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) $(2, 4)$

$y^2 = 18x$

$\left(\frac{9}{2}\right)^2 = 18x \rightarrow x = \frac{81}{4 \times 18} = \frac{9}{8}$

diff wrt x

Soln:-

$2y \cdot \frac{dy}{dx} = 18$

$\boxed{\frac{dy}{dx} = \frac{9}{y}}$ — ① → $dy = 2 \times dx$ given

$\boxed{\frac{dy}{dx} = 2}$

$\Rightarrow 2 = \frac{9}{y} \Rightarrow \boxed{y = 9/2}$

The slope of the normal to the curve $\rightarrow -\frac{1}{(dy/dx)}$

(a) $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is 0

(b) $x = 1 - a\sin\theta$, $y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$ is $\frac{a}{2b}$

(c) Both (a) and (b) are true

(d) Both (a) and (b) are not true

a) $\frac{dx}{d\theta} = a \cdot 3\cos^2\theta \times -\sin\theta$, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a \cdot 3\sin^2\theta \times \cos\theta}{a \cdot 3\cos^2\theta \times -\sin\theta} = \frac{-\sin\theta}{\cos\theta} = -\tan\theta$

$\frac{dy}{dx}\Big|_{\theta=\pi/4} = -\tan\frac{\pi}{4} = -1$

so slope of N $= \frac{-1}{-1} = 1$

dy/dx

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b\,2\cos\theta \times (-\sin\theta)}{-a\cos\theta}$

$\frac{dy}{dx} = \frac{+2b\sin\theta}{+a}\Big|_{\pi/2} = \frac{2b\sin\frac{\pi}{2}}{a}$

$= \frac{+2b}{a}$

slope normal $= +\frac{1}{+2b/a} = \frac{a}{2b}$

$= -\frac{a}{2b}$

Q. The interval in which the function $f(x) = \dfrac{4x^2+1}{x}$ is decreasing is :

(a) $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ (b) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(c) $(-1, 1)$ (d) $[-1, 1]$

$\Rightarrow f(x) = \dfrac{4x^2}{x} + \dfrac{1}{x}$

$f'(x) = 4 - \dfrac{1}{x^2}$

$f(x) \downarrow$

$f'(x) < 0$

$\boxed{4 - \dfrac{1}{x^2} < 0}$

$f'(x) = 0$

$4 - \dfrac{1}{x^2} = 0 \Rightarrow \dfrac{1}{x^2} = 4$

$x^2 = \dfrac{1}{4} \rightarrow \boxed{x = \pm\frac{1}{2}}$

$+ \quad - \quad +$

$-\infty \quad -1/2 \quad 1/2 \quad \infty$

$4 - \dfrac{1}{(-1)^2}$ $\quad 4 - \dfrac{1}{(1/4)^2}$ $\quad 4 - \dfrac{1}{(1)^2}$

$\rightarrow 4 - 1 \times 16 = -12$

The difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{2}$

(b) π

(c) $\frac{3\pi}{2}$

(d) $\frac{\pi}{4}$

$f'(x) = 2\cos 2x - 1$

let $f'(x) = 0$

$2\cos 2x - 1 = 0$

$\to 2\cos 2x = 1 \Rightarrow \cos 2x = \frac{1}{2}$

$\to \cos 2x = \cos \frac{\pi}{3}$ or $\cos -\frac{\pi}{3}$

$\cos\left(-\frac{\pi}{3}\right) \to \frac{1}{2}$

So $2x = \frac{\pi}{3}$ or $-\frac{\pi}{3}$ $\to x = \frac{\pi}{6}$ or $-\frac{\pi}{6}$

$-\frac{\pi}{2}$, $-\frac{\pi}{6}$, $\frac{\pi}{6}$, $\frac{\pi}{2}$

$\Rightarrow$ i) $f\left(-\frac{\pi}{2}\right) = \sin(-\pi) - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$

ii) $f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{3}\right) - \left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

iii) $f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) - \left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

iv) $f\left(\frac{\pi}{2}\right) = \sin \pi - \left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$

$\to$ Greatest val $= \frac{\pi}{2}$

$\to$ least value $= -\frac{\pi}{2}$

so diff $= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$

$= \pi$

On the interval [0, 1] the function $y = x^{25}(1-x)^{75}$ takes its maximum value at the point

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

So $f(x)$ takes max. value at $x = 1/4$

$y = x^{25}\cdot(1-x)^{75}$

$\frac{dy}{dx} = 25x^{24}\cdot(1-x)^{75} + 75(1-x)^{74}\times(-1)\times x^{25}$

$\frac{dy}{dx} = 25x^{24}\cdot(1-x)^{74}\left[1-x-3x\right] = 25x^{24}(1-x)^{74}(1-4x)$

$\rightarrow$ So $\frac{dy}{dx} = 0 = 25x^{24}(1-x)^{74}(1-4x) = 0$

$\boxed{x=0}$ $\boxed{x=1}$ $\boxed{x=1/4}$

$\rightarrow f(0) = 0\times(1-0)^{75} = 0$

$f(1) = 1\times(1-1)^{75} = 0$

$f(1/4) =$

$f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^{25}\times\left(1-\frac{1}{4}\right)^{75}$

$\left(\frac{1}{4}\right)^{25}\times\left(\frac{3}{4}\right)^{75}$

$(0.25)^{25}\times(0.75)^{75} > 0$

The slope of tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point $(2, -1)$ is

(a) $\frac{22}{7}$

(b) $\frac{6}{7}$

(c) $\frac{-6}{7}$

(d) -6

$\frac{dy}{dt} = 4t - 2$

$\frac{dx}{dt} = 2t + 3$

$\frac{42}{7} = 6$

So:- slope of Tangent $= \left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{4t-2}{2t+3}$ — ①

$\because$ given points are $(2,-1)$

$\therefore\ 2 = t^2 + 3t - 8$ — ②

$-1 = 2t^2 - 2t - 5$ — ③

$\Rightarrow 1 = 3t^2 + t - 13$

$\Rightarrow 3t^2 + t - 14 = 0$

$\Rightarrow 3t^2 + 7t - 6t - 14 = 0$

$\Rightarrow 3t^2 - 6t + 7t - 14 = 0$

$\Rightarrow 3t(t-2) + 7(t-2) = 0$

$(t-2)(3t+7) = 0$

$\boxed{t = 2}$ $\boxed{t = -7/3}$

So:- $\left.\frac{dy}{dx}\right|_{t=2} = \frac{4(t)-2}{2t+3}$

$= \frac{4(2)-2}{2\times 2+3} = \frac{6}{7}$

$\Rightarrow \frac{4\times\left(-\frac{7}{3}\right)-2}{2\times\left(-\frac{7}{3}\right)+3} = \frac{-34/3}{5/3}$

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The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

(c) $\left(0, \frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f(x) \to \uparrow$

$\boxed{f'(x) > 0}$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$\frac{1}{2}$	0

$1 > 0$

$\frac{\sqrt{3}}{2} > \frac{1}{2}$

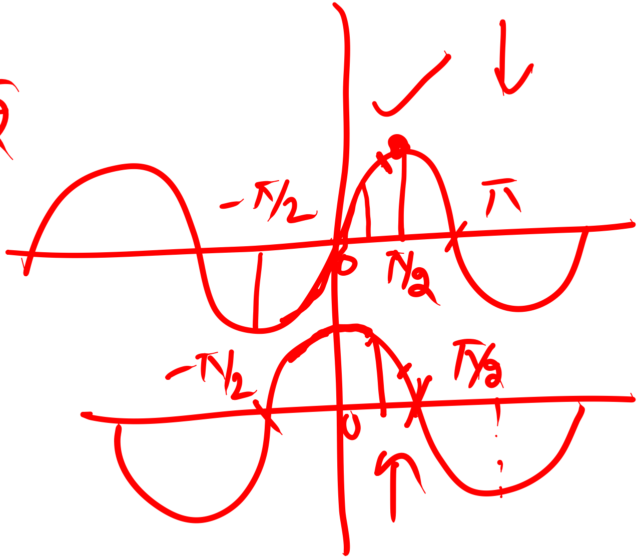
Sol^n $\boxed{f'(x) = \frac{1}{1+(\sin x+\cos x)^2} \times (\cos x - \sin x)}$

$\because f(x)$ is $\uparrow$ $\therefore f'(x) > 0$

$\left[\frac{\cos x - \sin x}{1+(\sin x+\cos x)^2} > 0\right] \to \cos x - \sin x > 0$

$1+(\sin x+\cos x)^2 \downarrow > 0$

$\boxed{\cos x > \sin x}$



$f(x) = -2x^3 - 9x^2 - 12x + 1$

$f'(x) = -6x^2 - 18x - 12$

Column-I	Column-II
A. $f(x) = x^2 - 2x + 5$ is	1. strictly decreasing in $(-\infty, -1)$ and strictly increasing in $(-1, -\infty)$.
B. $f(x) = 10 - 6x - 2x^2$ is	2. strictly increasing in $(-\infty, -9/2)$ and strictly decreasing in $\left(-\frac{9}{2}, \infty\right)$.
C. $f(x) = -2x^3 - 9x^2 - 12x + 1$ is	3. strictly decreasing in $(-\infty, -2)$ and $(-1, \infty)$ and strictly increasing in $(-2, -1)$
D. $f(x) = 6 - 9x - x^2$ is	4. strictly increasing in $(-\infty, -9/2)$ and strictly decreasing in $\left(-\frac{3}{2}, \infty\right)$
E. $(x+1)^3 (x-3)^3$ is	5. strictly increasing in $(1, 3)$ and $(3, \infty)$ and strictly decreasing in $(-\infty, -1)$ and $(-1, 1)$

Codes

	A	B	C	D	E
(a)	1	2	3	4	5
(b)	2	3	4	1	5
(c)	1	4	3	2	5
(d)	5	4	3	2	1

A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. The rate, at which its area is increasing when its radius is 3.2 cm, is

(a) $0.320\ \pi cm^2/s$ (b) $0.160\ \pi cm^2/s$

(c) $0.260\ \pi cm^2/s$ (d) $1.2\ \pi cm^2/s$

64, 5, 320, 2

0.320π

Sol: - $r = 3\,cm$, $\frac{dr}{dt} = 0.05$ cm/sec.

find $\frac{dA}{dt} = ?$ → at $r = 3.2$

$\because A = \pi r^2$ → $\left.\frac{dA}{dt}\right|_{3.2} = \pi 2r \times \frac{dr}{dt}$

$= \pi \times 2 \times 3.2 \times 0.05$

If $f(x) = \cos x$, $g(x) = \cos 2x$, $h(x) = \cos 3x$ and $I(x) = \tan x$, then which of the following option is correct?

(a) $f(x)$ and $g(x)$ are strictly decreasing in $(0, \pi/2)$

(b) $h(x)$ is neither increasing nor decreasing in $(0, \pi/2)$

(c) $I(x)$ is strictly increasing in $(0, \pi/2)$

(d) All are correct

$I(x) = \tan x$

$I'(x) = \sec^2 x \;\; \xrightarrow{} > 0$

$\cos x^2 \quad (0, \pi)$

$(0, \pi/2)$

$(1)^2, \left(\frac{\sqrt{3}}{2}\right)^2, \left(\frac{1}{\sqrt{2}}\right)^2, \left(\frac{1}{2}\right)^2, 0$

1	2	3	4	5	6	7	8	9	10
A	D	A	B	B	B	B	C	A	D

Q. Find approx. change in volume V of a cube of side x metres caused by increasing the side by 2%.

Sol^n. Cube :→ $\boxed{V = x^3}$

$\frac{dv}{dx} = 3x^2$

side $= x$ m → given → ↑ side by 2%

so side become $= 0.02x$

$\boxed{\Delta x = 0.02x}$

$\Delta V = ?$

$\because \frac{\Delta V}{\Delta x} = \frac{dv}{dx}$

$\Delta V = \Delta x \times \frac{dv}{dx} = 0.02 \times 3x^2$

$= 0.06x^2 \ m^3$ Ans

ΔV

$V = x^3$

$10m = 1000 cm$

$\times \frac{2}{100}$

$20 cm$ ↑

$\boxed{V = 1000 m^3}$

2%

$\boxed{V = 1000 \times 10^6 cm^3}$

$\boxed{1020} cm$

$V = (1020)^3$

$(102)^3 \times 10^3$