A point on the parabola $y^2=18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) $\left(\frac{9}{8}, \frac{9}{2}\right)$ (b) $(2, -4)$

(c) $\left(\frac{-9}{8}, \frac{9}{2}\right)$ (d) $(2, 4)$

$y^2 = 18x$

$\left(\frac{9}{2}\right)^2 = 18x \rightarrow x = \frac{81}{4 \times 18}$

$\frac{9}{8}$

diff wrt x

Soln:-

$2y \cdot \frac{dy}{dx} = 18$

$\boxed{\frac{dy}{dx} = \frac{9}{y}}$ — ① $\rightarrow$ given $dy = 2 \times dx$

$\boxed{\frac{dy}{dx} = 2}$

$\Rightarrow 2 = \frac{9}{y} \Rightarrow \boxed{y = 9/2}$

The slope of the normal to the curve $\rightarrow -\frac{1}{(dy/dx)}$

(a) $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$ is 0

(b) $x = 1 - a\sin\theta$, $y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}$ is $\frac{a}{2b}$

(c) Both (a) and (b) are true

(d) Both (a) and (b) are not true

dy/dx

a) $\frac{dx}{d\theta} = a \cdot 3\cos^2\theta \times -\sin\theta$, $\frac{dy/d\theta}{dx/d\theta} = \frac{a \cdot 3\sin^2\theta \times \cos\theta}{a \cdot 3\cos^2\theta \times -\sin\theta}$

$= \frac{-\sin\theta}{\cos\theta} = -\tan\theta$

$\frac{dy}{dx}\Big|_{\theta=\pi/4} = -\tan\pi/4 = -1$

so slope of N $= -\frac{1}{-1} = 1$

$\frac{dy}{d\theta} = \frac{b\,2\cos\theta \times -\sin\theta}{dx/d\theta}$

$\frac{dx}{d\theta} = -a\cos\theta$

$\frac{dy}{dx} = \frac{+2b\sin\theta}{+a}\Big|_{\pi/2} = \frac{2b\sin\frac{\pi}{2}}{a}$

$= \frac{+2b}{a}$

slope normal $= +\frac{1}{+2b/a} = \frac{a}{2b}$

$= -\frac{a}{2b}$

Q. The interval in which the function $f(x) = \dfrac{4x^2+1}{x}$ is decreasing is : $\Rightarrow f(x) = \dfrac{4x^2}{x} + \dfrac{1}{x}$

(a) $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$ $f'(x) = 4 - \dfrac{1}{x^2}$ (b) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

(c) $(-1, 1)$ $f(x) \to \downarrow$ (d) $[-1, 1]$

$\therefore f'(x) < 0$

$\boxed{4 - \dfrac{1}{x^2} < 0}$

$\to f'(x) = 0$

$4 - \dfrac{1}{x^2} = 0 \Rightarrow \dfrac{1}{x^2} = 4$

$x^2 = \dfrac{1}{4} \to \boxed{x = \pm\dfrac{1}{2}}$

$+ \quad - \quad +$

$-\infty \quad -1/2 \quad 1/2 \quad \infty$

$4 - \dfrac{1}{(-1)^2}$ $\quad 4 - \dfrac{1}{(1/4)^2}$ $\quad 4 - \dfrac{1}{(1)^2}$

$\to 4 - \dfrac{1 \times 16}{1}$

$= -12$

The difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ is

(a) $\frac{\pi}{2}$

(b) π

(c) $\frac{3\pi}{2}$

(d) $\frac{\pi}{4}$

$f'(x) = 2\cos 2x - 1$

let $f'(x) = 0$

$2\cos 2x - 1 = 0$

$\to 2\cos 2x = 1 \Rightarrow \cos 2x = \frac{1}{2}$

$\to \cos 2x = \cos \pi/3$ or $\cos -\pi/3$

$\cos(-\pi/3) = \frac{1}{2}$

So $2x = \frac{\pi}{3}$ or $-\frac{\pi}{3} \to x = \frac{\pi}{6}$ or $-\pi/6$

$-\pi/2 \quad -\pi/6 \quad \pi/6 \quad \pi/2$

$\Rightarrow$ i) $f\left(-\frac{\pi}{2}\right) = \sin(-\pi) - \left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$

ii) $f\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{3}\right) - \left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} + \pi/6$

iii) $f(\pi/6) = \sin(\pi/3) - (\pi/6) = \frac{\sqrt{3}}{2} - \pi/6$

iv) $f(\pi/2) = \sin \pi - (\pi/2) = -\pi/2$

$\to$ Greatest val $= \pi/2$

$\to$ least value $= -\pi/2$

so diff $= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$

$= \pi$

On the interval [0, 1] the function $y = x^{25}(1-x)^{75}$ takes its maximum value at the point

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

So $f(x)$ takes max. value at $x = 1/4$

$y = x^{25}\cdot(1-x)^{75}$

$\frac{dy}{dx} = 25x^{24}\cdot(1-x)^{75} + 75(1-x)^{74}\times(-1)\times x^{25}$

$\frac{dy}{dx} = 25x^{24}\cdot(1-x)^{74}[1-x-3x] = 25x^{24}(1-x)^{74}(1-4x)$

$\rightarrow$ So $\frac{dy}{dx} = 0 = 25x^{24}(1-x)^{74}(1-4x) = 0$

$\boxed{x=0}$ $\boxed{x=1}$ $\boxed{x=1/4}$

$\rightarrow f(0) = 0\times(1-0)^{75} = 0$

$f(1) = 1\times(1-1)^{75} = 0$

$f(1/4) =$

$f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^{25}\times\left(1-\frac{1}{4}\right)^{75}$

$\left(\frac{1}{4}\right)^{25}\times\left(\frac{3}{4}\right)^{75}$

$(0.25)^{25}\times(0.75)^{75} > 0$

The slope of tangent to the curve

$x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point

(2, – 1) is

(a) $\frac{22}{7}$ $\qquad$ (b) $\frac{6}{7}$

(c) $\frac{-6}{7}$ $\qquad$ (d) – 6

$\frac{dy}{dt} = 4t - 2$

$\frac{dx}{dt} = 2t + 3$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

So:- slope of Tangent $= \left.\frac{dy}{dx}\right|_{(2,-1)} = \frac{4t-2}{2t+3}$ — (1)

∵ given points are (2, –1)

∴ $2 = t^2 + 3t - 8$ — (2)

$-1 = 2t^2 - 2t - 5$ — (3)

$\Rightarrow 1 = 3t^2 + t - 13$

$\Rightarrow 3t^2 + t - 14 = 0$

$\Rightarrow 3t^2 + 7t - 6t - 14 = 0$

$\Rightarrow 3t^2 - 6t + 7t - 14 = 0$

$\Rightarrow 3t(t-2) + 7(t-2) = 0$

$(t-2)(3t+7) = 0$

$t = 2$ $\qquad$ $t = -7/3$

So:- $\left.\frac{dy}{dx}\right|_{t=2} = \frac{4(t)-2}{2t+3}$

$= \frac{4(2)-2}{2\times2+3} = \frac{6}{7}$

$\Rightarrow \frac{4\times(-\frac{7}{3})-2}{2\times(-\frac{7}{3})+3} = \frac{-34/3}{5/3}$

The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(b) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

(c) $\left(0, \frac{\pi}{2}\right)$

(d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Column-I	Column-II
A. $f(x) = x^2 - 2x + 5$ is	1. strictly decreasing in $(-\infty, -1)$ and strictly increasing in $(-1, -\infty)$.
B. $f(x) = 10 - 6x - 2x^2$ is	2. strictly increasing in $(-\infty, -9/2)$ and strictly decreasing in $\left(-\frac{9}{2}, \infty\right)$.
C. $f(x) = -2x^3 - 9x^2 - 12x + 1$ is	3. strictly decreasing in $(-\infty, -2)$ and $(-1, \infty)$ and strictly increasing in $(-2, -1)$
D. $f(x) = 6 - 9x - x^2$ is	4. strictly increasing in $(-\infty, -9/2)$ and strictly decreasing in $\left(-\frac{3}{2}, \infty\right)$
E. $(x+1)^3 (x-3)^3$ is	5. strictly increasing in $(1, 3)$ and $(3, \infty)$ and strictly decreasing in $(-\infty, -1)$ and $(-1, 1)$

Codes

	A	B	C	D	E
(a)	1	2	3	4	5
(b)	2	3	4	1	5
(c)	1	4	3	2	5
(d)	5	4	3	2	1

A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. The rate, at which its area is increasing when its radius is 3.2 cm, is

(a) $0.320\ \pi cm^2/s$ (b) $0.160\ \pi cm^2/s$

(c) $0.260\ \pi cm^2/s$ (d) $1.2\ \pi cm^2/s$

If $f(x) = \cos x$, $g(x) = \cos 2x$, $h(x) = \cos 3x$ and $I(x) = \tan x$, then which of the following option is correct?

(a) $f(x)$ and $g(x)$ are strictly decreasing in $(0, \pi/2)$

(b) $h(x)$ is neither increasing nor decreasing in $(0, \pi/2)$

(c) $I(x)$ is strictly increasing in $(0, \pi/2)$

(d) All are correct

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1	2	3	4	5	6	7	8	9	10
A	D	A	B	B	B	B	C	A	D