AOD

Ex:- prove that the curve $x = y^2$ (1) & $xy = k$ (2) cut at right angles if $8k^2 = 1$

Sol^n:- ∵ both curve cut each other ∴ intersection point of both curve will be

$\Rightarrow x = y^2$ — (1) & $xy = k$ — (2)

from (1) & (2) :- $y^2 \cdot y = k \Rightarrow y^3 = k$

$\Rightarrow \boxed{y = k^{1/3}}$ & put y in eq. (1) $\rightarrow \boxed{x = k^{2/3}}$

So intersection point $(x, y) \rightarrow (k^{2/3}, k^{1/3}) \rightarrow P$

Now $\boxed{\text{from eq}^n \text{ (1)}}$:- $x = y^2$

diff $\rightarrow 1 = 2y \cdot \frac{dy}{dx} \Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{2y}}$

So :- Slope of Tangent of curve (1) on point P:

$$\left.\frac{dy}{dx}\right|_{(k^{2/3},\, k^{1/3})} = \frac{1}{2 \cdot k^{1/3}} = M_1$$

from eq^n (2) :- $xy = k \rightarrow$ diff.

$\Rightarrow x \cdot \frac{dy}{dx} + y(1) = 0$

∴ $\frac{dy}{dx} = -y/x \rightarrow$ so slope of Tangent of curve (2) on point P.

$$\left.\frac{dy}{dx}\right|_{(k^{2/3},\, k^{1/3})} = \frac{-k^{1/3}}{k^{2/3}} = -k^{1/3} \cdot k^{-2/3} = -k^{-1/3}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{(k^{2/3},\, k^{1/3})} = \frac{-1}{k^{1/3}} = M_2}$$

Now :- ∵ Both curve cut each other at 90° ∴ The Tangent of both curves are perpendicular to each other.

So :- $M_1 \cdot M_2 = -1 \Rightarrow \frac{1}{2 \cdot k^{1/3}} \cdot \frac{\cancel{-}1}{k^{1/3}} = \cancel{-}1 \Rightarrow \frac{1}{2 \cdot k^{2/3}} = 1$

$\Rightarrow 1 = 2 \cdot k^{2/3} \rightarrow$ on Cubeing both sides - $1 = (2k^{2/3})^3$

$\Rightarrow \boxed{1 = 8 \cdot k^2}$ H.P.

AOD

Ex:- $y = x^2 - 2x + 7 \rightarrow$ Eq. of Tangent $\rightarrow (y - y_1) = M_T (x - x_1)$

a) $\parallel$ to the line $2x - y + 9 = 0$

Solⁿ:- $\because$ given line: $2x - y + 9 = 0$

$\Rightarrow \boxed{y = 2x + 9} \rightarrow \boxed{y = mx + c}$

$\therefore \boxed{m = 2} \rightarrow$ slope of line $\boxed{2x - y + 9 = 0}$

$\because$ it is given that the tangent of curve is $\parallel$ to line $2x - y + 9 = 0$

$\therefore$ Slope of Tangent = Slope of line = 2 — (1)

Now from curve: $y = x^2 - 2x + 7$

diff. $\rightarrow \frac{dy}{dx} = 2x - 2 =$ Slope of Tangent — (2)

from eqⁿ (1) & (2): $2x - 2 = 2 \Rightarrow \boxed{x = \frac{4}{2} = 2}$

then $y = (2)^2 - 2(2) + 7 = 7 \Rightarrow \boxed{y = 7}$

So the eqn. of Tangent:-

$\therefore (y - 7) = 2(x - 2) \Rightarrow y - 7 = 2x - 4$

$\Rightarrow \boxed{2x - y + 3 = 0} \rightarrow$ A

$$\begin{array}{r} 277 \\ 60 \\ \hline 217 \end{array}$$

b) Which is $\perp$ to the line $5y - 15x = 13$

Solⁿ:- $\because$ line: $5y - 15x = 13$

$\rightarrow y = \frac{3\cancel{15}x}{\cancel{5}} + \frac{13}{5} \Rightarrow$ on Comparing $\boxed{m = 3} = M_L$

$\because$ line & tangent of curve are $\perp$.

$\therefore M_L \times M_T = -1 \Rightarrow \boxed{M_T = \frac{-1}{3}}$ = slope of Tangent

$\rightarrow \because$ from given curve: $\boxed{\frac{dy}{dx} = 2x - 2 = \frac{-1}{3}}$

$\Rightarrow 2x = 2 - \frac{1}{3} = \frac{5}{3} \Rightarrow \boxed{x = \frac{5}{6}} \rightarrow$ put $x = 5/6$ in y:-

$y = \frac{25}{36} - \frac{10}{6} + \frac{7}{1} = \frac{25 - 60 + 252}{36} = \boxed{\frac{217}{36} = y}$

So The eq. of Tangent $\rightarrow \left(y - \frac{217}{36}\right) = -\frac{1}{3}\left(x - \frac{5}{6}\right)$ Ans

$= \boxed{36y + 12x - 227 = 0}$

AOD

Ex:- Find Eq. of Tangent & normal of curve $y = |x^2 - |x||$ at $\boxed{x = -2}$

Sol^n:- $\because y = |x^2 - |x||$

Here we have to find the eq. of tangent & normal at $\boxed{x = -2}$

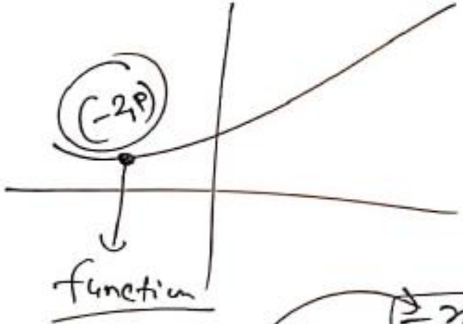
$\therefore \quad y = |x^2 - (-x)|$

$y = |x^2 + x|$

$\boxed{y = x^2 + x}$ → H.W.

$|x^2 - x|$

$x^2 - x$



$|x| \to -x \to x < 0$

$|x| \to x \to x > 0$

$x = -2 \to -(-2) = 2$

$|x| = 2$