

Which of the following function is

$$\underline{\sec^2 \eta > 0}$$

decreasing on  $\left(0, \frac{\pi}{2}\right)$ ?

(a)  $\sin 2x$

(b)  $\tan x$

(c)  $\cos x$

(d)  $\cos 3x$

$$f(\eta) = \cos \eta$$

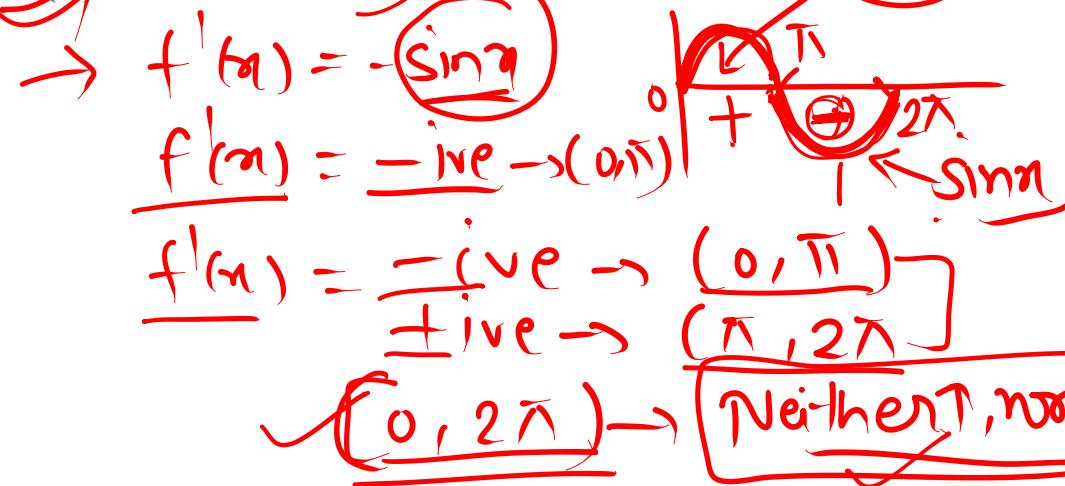
$$f'(\eta) = -\sin \eta$$



$$\underline{f'(\eta) < 0}$$

If  $f(x) = \cos x$ , then  $f'(x) = -\sin x$

- (a)  $f(x)$  is strictly decreasing in  $(0, \pi)$
- (b)  $f(x)$  is strictly increasing in  $(0, 2\pi)$  ( $\pi, 2\pi$ )
- (c)  $f(x)$  is neither increasing nor decreasing in  $(\pi, 2\pi)$  ( $0, 2\pi$ )
- (d) All the above are correct



The volume V and depth x of water in a vessel are connected by the relation  $V = 5x - \frac{x^2}{6}$  and the volume of water is increasing, at the rate of  $5 \text{ cm}^3/\text{sec}$ , when  $x = 2 \text{ cm}$ . The rate at which the depth of water is increasing, is

(a)  $\frac{5}{18} \text{ cm/sec}$

(b)  $\frac{1}{4} \text{ cm/sec}$

(c)  $\frac{5}{16} \text{ cm/sec}$

(d) None of these

$$\begin{aligned} V &= 5x - \frac{x^2}{6} \\ \therefore \frac{dV}{dt} &= 5 \text{ cm}^3/\text{s} \end{aligned}$$

$$\frac{dV}{dt} = 5 \rightarrow \frac{dx}{dt} = ?$$

Shape → No info

$$\frac{dV}{dx} = 5 - \frac{2x}{6} = 5 - \frac{x}{3}$$

$$\begin{aligned} \frac{dV}{dt} &= \frac{5}{dt} - \frac{2x}{6} \times \frac{dx}{dt} \\ \Rightarrow 5 &= \frac{dx}{dt} \left[ 5 - \frac{x}{3} \right] \end{aligned}$$

$$\frac{dx}{dt} = \frac{5}{15-x} = \frac{5 \times 3}{15-x}$$

$$\begin{aligned} x &= 2 \\ \frac{dx}{dt} &= \frac{15}{15-2} = \frac{15}{13} \text{ cm/sec} \end{aligned}$$

Ans

A ladder is resting with the wall at an angle of  $30^\circ$ . A man is ascending the ladder at the rate of  $3 \text{ ft/sec}$ . His rate of approaching the wall is → according to fig. →

$$\therefore R = 3 \text{ ft/sec}$$

(a)  $3 \text{ ft/sec}$

(b)  $\frac{3}{2} \text{ ft/sec}$

(c)  $\frac{3}{4} \text{ ft/sec}$

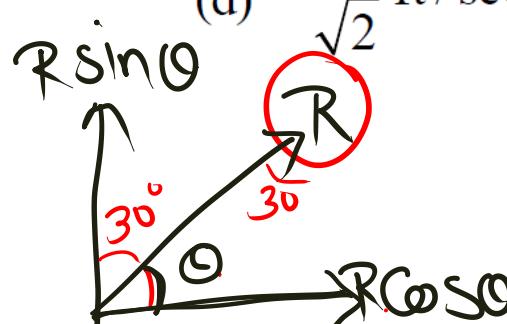
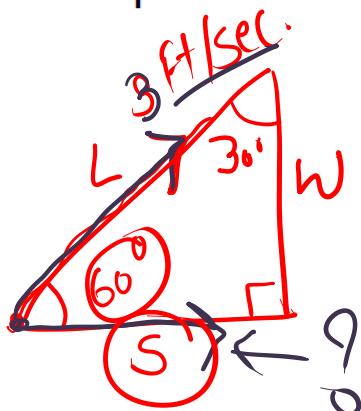
(d)  $\frac{3}{\sqrt{2}} \text{ ft/sec}$

$$S = R \cos \theta$$

$$S = 3 \times \cos 60^\circ$$

$$= 3 \times \frac{1}{2}$$

$$= 1.5$$



What is the interval in which the function

$$f(x) = \sqrt{9 - x^2}$$
 is increasing? ( $f(x) > 0$ )

(a)  $0 < x < 3$

(b)  $-3 < x < 0$

(c)  $0 < x < 9$

(d)  $-3 < x < 3$

Finding

$$-3 < x < 3$$

$$f'(x) > 3 < x < 0$$

$$\rightarrow f(x) = \sqrt{9 - x^2} \quad [x < 0]$$

$$f'(x) = \frac{1}{2\sqrt{9-x^2}} \cdot -2x > 0$$

$$\frac{-2x}{2\sqrt{9-x^2}} > 0 \quad [x < 0]$$

$$\sqrt{9-x^2} \rightarrow \text{Real}$$

$$9 - x^2 > 0 \quad \frac{x^2 - 9}{(x-3)(x+3)} < 0 \\ x = 3, -3$$

What is the slope of the normal at the point

( $at^2$ ,  $2at$ ) of the parabola  $y^2 = 4ax$ ?

(a)  $\frac{1}{t}$

$\Rightarrow y^2 = 4ax$

(b)  $t$

(d)  $-\frac{1}{t}$

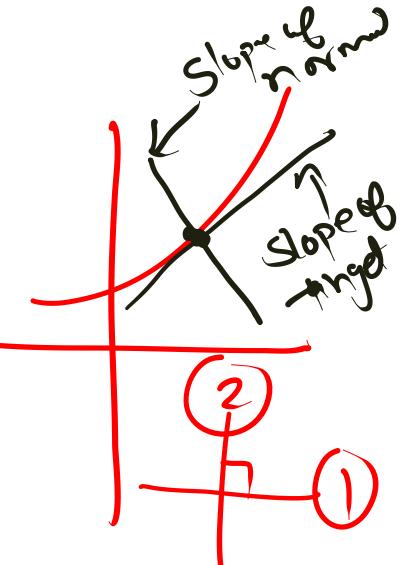
(c)  $-t$

Slope of tangent =  $\frac{dy}{dx}$

Slope of normal =  $-(\frac{dx}{dy})$

$\Rightarrow 2y \left( \frac{dy}{dx} \right) = 4a \Rightarrow \frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$

$\frac{dy}{dx} = \frac{4a}{2y} = \frac{2a}{y}$



So slope of normal  
 $= -\left(\frac{dx}{dy}\right) = -\left(\frac{y}{2a}\right)$   
 at  $(at^2, 2at)$   
 $\therefore \left(\frac{dx}{dy}\right) = -\left(\frac{2at}{2a}\right) = -t$

$m_1 \cdot m_2 = -1$

$\frac{dy}{dx} \cdot m_2 = -1$

$m_2 = -\frac{1}{\frac{dy}{dx}} = -\frac{1}{\frac{2a}{y}}$

$= -\left(\frac{dy}{dx}\right)$

The maximum value of the function

$$y = -x^2$$
 in the interval  $[-1, 1]$  is

(a) 0

(b) 2

(c) 8

(d) 9

$$y = -x^2$$

$$-x^2 \geq 0$$

$$-x^2 < 0 \quad \underline{\max \rightarrow 0}$$

A stone is dropped into a quiet lake and waves moves in circles at the speed of 5 cm/s. If at a instant, the radius of the circular wave is 8 cm, then the rate at which enclosed area is increasing, is

- (a)  $20\pi \text{ cm}^2/\text{s}$
- (b)  $40\pi \text{ cm}^2/\text{s}$
- (c)  $60\pi \text{ cm}^2/\text{s}$
- (d)  ~~$80\pi \text{ cm}^2/\text{s}$~~

$$\frac{dr}{dt} = 5 \text{ cm/sec}$$

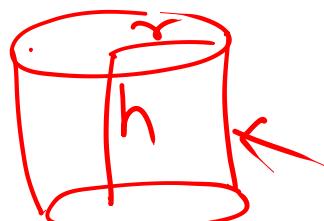
$$r = 8 \rightarrow \frac{dA}{dt} \uparrow$$

$$\frac{d(\pi r^2)}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt} = \pi \times 2 \times 8 \times 5$$

$$= 80\pi \text{ cm}^2/\text{s}$$

The radius of a cylinder is increasing at the rate of 3 m/s and its altitude is decreasing at the rate of 4 m/s. The rate of change of volume when radius is 4 m and altitude is 6m, is

- (a)  $20\pi \text{ m}^3/\text{s}$       (b)  $40\pi \text{ m}^3/\text{s}$   
 (c)  $60\pi \text{ m}^3/\text{s}$       (d) ~~None of these~~



$$\frac{dr}{dt} = 3 \text{ m/s}, \frac{dh}{dt} = -4 \text{ m/sec}$$

$$\frac{dv}{dt} = ? \rightarrow \text{at } r=4\text{m} \& h=6\text{m}$$

$$\frac{d}{dt} [\Delta r^2 h] = \partial \frac{d}{dt} (r^2 h)$$

$$\begin{aligned} \frac{dv}{dt} &= \frac{\partial V}{\partial r} \frac{dr}{dt} + \frac{\partial V}{\partial h} \frac{dh}{dt} \\ \frac{dV}{dt} &= (4^2) \cdot (-4) + 6 \times 2 \times 4 \times 3 \\ &= [-64 + 144] \\ &= \cancel{\underline{\underline{\pi 80}}} \text{ m}^3/\text{sec.} \end{aligned}$$



If sum of two numbers is 3, the maximum value of the product of first and the square of second is

(a) 4

$$\underline{x+y=3}$$

(b) 3

(c) 2

$$\rightarrow \underline{x(y^2)=?}$$

(d) 1

$\rightarrow$

x	y	3	$\underline{xy^2}$
1	2	3	4
0	3	3	0 ✓
3	0	3	0 ✓
2	-1	3	2 ✓

1	2	3	4	5	6	7	8	9	10
C	A	D	B	B	C	A	D	D	A