AOD #1

Ex1:- $y = [x(x-2)]^2 \rightarrow x = ?$ for ↑ fn.

Soln: $f(x) = [x^2 - 2x]^2$

$\therefore f'(x) = 2(x^2 - 2x)(2x - 2)$

$f'(x) = 2 \times 2(x^2 - 2x)(x - 1)$

$f'(x) = 4x(x-2)(x-1)$

Now:- $f'(x) = 0 = 4x(x-2)(x-1)$

Here $\boxed{x = 0}$ & $\boxed{x = 2}$ & $\boxed{x = 1}$

so here we get value of $x = 0, 1, 2$ therefore domain of $f(x)$ is divided into 4 intervals.

$\Rightarrow$

intervals	$f'(x)$	Nature of $f(x)$
$(-\infty, 0)$	-ive	↓
$(0, 1)$	+	↑
$(1, 2)$	-ive	↓
$(2, \infty)$	+	↑

So for $(0, 1)$ & $(2, \infty)$ the $f(x)$ is increasing Ans

AOD

Ex:- $(0, \infty) \rightarrow f(x) = \log x \rightarrow$ always $\uparrow$.

Sol^n:- $\because f(x) = \log x$

$\therefore f'(x) = \dfrac{1}{x}$, $\boxed{x \neq 0}$

0 ——————————— ∞

$\because$ Here the given interval is $(0, \infty)$ during which the function $f'(x)$ is always greater than zero.

$\therefore f'(x) = \dfrac{1}{x} > 0$ in $(0, \infty)$

So:- $f(x)$ is always increasing. Ans

AOD

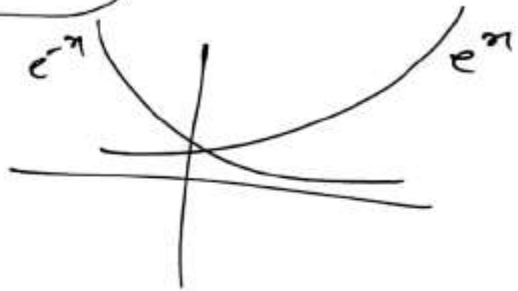
Ex:- $y = x^2 \cdot e^{-x}$, then interval in which $\boxed{y \text{ increase}}$ wrt x.

Sol^n:- $y = f(x) = x^2 \cdot e^{-x}$

$\therefore \Rightarrow f'(x) = x^2 \cdot e^{-x}(-1) + e^{-x} \cdot 2x$

$= e^{-x}(-x^2 + 2x)$

$$\boxed{f'(x) = x \cdot e^{-x}(2-x)}$$



$\because$ we have to find the interval in which $f(x)$ is $\uparrow$.

$\therefore \quad f'(x) > 0$

$x \cdot e^{-x}(2-x) > 0$

So:- $x > 0$, $x < 2$ $\rightarrow$ Then in $0 < x < 2 = (0,2)$ interval

$f(x)$ always $\uparrow$.

AOD

Ex:- prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is increasing fun. of θ in $\left[0, \frac{\pi}{2}\right]$

Sol^n.- $f(x) = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$

0 —— $\pi/2$

$\therefore f'(x) = \frac{(2+\cos\theta)4\cdot\cos\theta - 4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2} - 1$

$\Rightarrow f'(x) = \frac{8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$

$$\boxed{f'(x) = \frac{8\cos\theta + 4}{(2+\cos\theta)^2} - 1}$$

Now: $f'(x) = 0 = \frac{8\cos\theta+4}{(2+\cos\theta)^2} - 1 \Rightarrow \frac{8\cos\theta+4}{(2+\cos\theta)^2} = 1$

$= 8\cos\theta + \cancel{4} = \cancel{4} + 4\cos\theta + \cos^2\theta$

$\Rightarrow 4\cos\theta - \cos^2\theta = 0 \Rightarrow \cos\theta(4 - \cos\theta) = 0$

$\Rightarrow \cos\theta = 0$ | $4 - \cos\theta = 0$

$\theta = \pi/2$ | $\cos\theta = 4$ (N.P.)

→ So here in interval $[0, \pi/2]$ $f'(x)$ is always greater than zero. Therefor $f(x)$ is increasing $[0, \pi/2]$

H.P.

AOD

Ex:- Which of the following are strictly ↓ on $\left(0, \frac{\pi}{2}\right)$?

a) $\cos x$ ✓ b) $\cos 2x$ c) $\cos 3x$ D) $\tan x$

Sol^n: (b) $f(x) = \cos 2x$

$f'(x) = -\sin(2x) \cdot 2$

$\because f'(x) = 0 \Rightarrow -2\sin 2x = 0$

$\Rightarrow \sin 2x = 0$

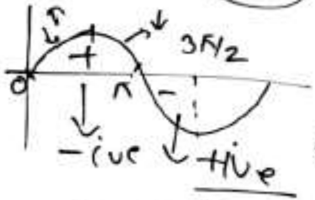
$\Rightarrow \boxed{\sin 2x = \sin \pi}$

$\Rightarrow \boxed{2x = \pi}$

$x = \frac{\pi}{2}$

$0 < x < \frac{\pi}{2}$

$0 < 3x < \frac{3\pi}{2}$



$\because$ given interval is $\left(0, \frac{\pi}{2}\right) \Rightarrow 0 < x < \frac{\pi}{2}$

$\boxed{0 < 2x < \pi}$

Here when $0 < 2x < \pi \Rightarrow f'(x) < 0$

So: $f(x) = \cos 2x$ is ↓ in $\left(0, \frac{\pi}{2}\right)$ Ans

(C) $f(x) = \cos 3x \Rightarrow f'(x) = -\sin 3x \cdot 3$

$\Rightarrow f'(x) = 0 = -3\sin 3x$

$\Rightarrow \sin 3x = 0$

$\Rightarrow \sin 3x = \sin \pi$

$\Rightarrow \boxed{x = \frac{\pi}{3}}$

0 (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$

−ive +ive

Here in $\left(0, \frac{\pi}{3}\right) \to f'(x) < 0$

& $\left(\frac{\pi}{3}, \frac{\pi}{2}\right) \to f'(x) > 0$

neither ↑ nor ↓. Ans