Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}.

(a) R is reflexive and symmetric but not transitive φ

(b) R is reflexive and transitive but not symmetric

(c) R is symmetric and transitive but not reflexive φ

(d) R is equivalence relation φ

(1, 2) (1 3) (3 2)

R

a, b ∈ R b, c ∈ R

a, c ∈ R

Let $f(x) = \dfrac{ax+b}{cx+d}$. Then $fof(x) = x$ provided that

(a) $d = -a$

(b) $d = a$

(c) $a = b = c = d = 1$

(d) $a = b = 1$

$f(x)$

$\Rightarrow f(f(x)) = x \Rightarrow f\left(\dfrac{ax+b}{cx+d}\right) = x$

$$\Rightarrow \frac{a\left[\dfrac{ax+b}{cx+d}\right] + b}{c\left[\dfrac{ax+b}{cx+d}\right] + d} = x$$

$$\Rightarrow \frac{a^2x + ab + bcx + bd}{acx + cb + cdx + d^2} = x$$

$$\Rightarrow \frac{x[a^2 + bc] + [ab + bd]}{x[ac + cd] + [cb + d^2]} = x$$

Solve

LHS → RHS

$\boxed{d = -a}$ $\quad d^2 = a^2$

$$\frac{x[a^2 + bc] + \cancel{ab} - \cancel{ab}}{x[\cancel{ac} + (\cancel{-ac})] + cb + a^2} = x$$

$$\frac{x[\cancel{a^2 + bc}]}{\cancel{cb + a^2}} = x$$

$\boxed{x = x}$

$\boxed{fof(x) = x}$

Let $f : N \to R$ be the function defined by $f(x) = \frac{2x-1}{2}$ and $g : Q \to R$ be another function defined by $g(x) = x+2$. Then $(gof)\left(\frac{3}{2}\right)$ is

(a) 1

(b) 0

(c) $\frac{7}{2}$

(d) 3

$3/2$

Sol^n: $gof(x) \to g[f(x)] = g\left[\frac{2x-1}{2}\right]$

$\Rightarrow \frac{2x-1}{2} + 2 = \frac{2x-1+4}{2} = \frac{2x+3}{2}$

$\Rightarrow gof(3/2) = \frac{\cancel{2} \times \frac{3}{\cancel{2}} + 3}{2} = \frac{6}{2} = 3$

If the binary operation * is defined on the set Q^+ of all positive rational numbers by $\boxed{a * b = \frac{ab}{4}}$. Then $3 * \left(\left(\frac{1}{5} * \frac{1}{2}\right)\right)$ is equal to

(a) $\frac{3}{160}$ (b) $\frac{5}{160}$

(c) $\frac{3}{10}$ (d) $\frac{3}{40}$

$$\frac{\frac{1}{5} \times \frac{1}{2}}{4}$$

$$3 \times \frac{1}{40} = \frac{3 \times \frac{1}{40}}{4}$$

$$\frac{3}{40} \times \frac{1}{4} = \frac{3}{160}$$

The inverse of the function $f(x)=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}+2$ is

(a) $\log_e\left(\frac{x-3}{x-1}\right)^{1/2}$ (b) $\log_e\left(\frac{x-1}{3-x}\right)^{1/2}$

(c) $\log_e\left(\frac{x+2}{x-3}\right)^{1/2}$ (d) $\log_e\left(\frac{x+1}{x-2}\right)^{1/2}$

inverse → $x = y$

given

Sol: $y-2=\frac{e^{x}-\frac{1}{e^{x}}}{e^{x}+\frac{1}{e^{x}}} \Rightarrow y-2=\frac{e^{2x}-1}{e^{2x}+1}$

$\Rightarrow y\cdot e^{2x}+y-2\cdot e^{2x}-2=e^{2x}-1$

$\Rightarrow y\cdot e^{2x}-2e^{2x}-e^{2x}=-1+2-y$

$\Rightarrow ye^{2x}-3e^{2x}=1-y$

$\rightarrow e^{2x}(y-3)=1-y$

$\rightarrow \boxed{e^{2x}=\frac{1-y}{y-3}}$

$\rightarrow \log e^{2x}=\log\left(\frac{1-y}{y-3}\right)$

$2x\cdot\log e=\log\left(\frac{1-y}{y-3}\right) \Rightarrow x=\frac{1}{2}\log\left(\frac{1-y}{y-3}\right)$

$x=\cancel{\frac{1}{2}}\left[\log_e\left(\frac{1-y}{y-3}\right)\right]^{1/2}$ → inverse

So: inverse of $y \Rightarrow \log_e\left(\frac{1-x}{x-3}\right)^{1/2}$

$\log_e\left(\frac{x-1}{3-x}\right)^{1/2}$ Ans

If $f : R \to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$,

then fof (x) is

(a) $x^{\frac{1}{3}}$ $fof(x) = ?$

(b) x^3

(c) x $\Rightarrow f[f(x)]$

(d) $(3 - x^3)$

$\Rightarrow f[(3-x^3)^{1/3}] \to [3 - \{(3-x^3)^{1/3}\}^3]^{1/3}$

$\boxed{f(x)}$

$(3 - 3 + x^3)^{1/3}$

$(x^3)^{1/3} = x$

The value of x obtained from the equation

$$\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+\gamma \end{vmatrix} = 0 \text{ will be}$$

(a) 0 and $-(\alpha+\beta+\gamma)$

(b) 0 and $\alpha+\beta+\gamma$

(c) 1 and $(\alpha-\beta-\gamma)$

(d) 0 and $\alpha^2+\beta^2+\gamma^2$

$x = -(\alpha+\beta+\gamma)$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} x+\alpha+\beta+\gamma & \beta & \gamma \\ x+\alpha+\beta+\gamma & x+\beta & \alpha \\ x+\alpha+\beta+\gamma & \beta & x+\gamma \end{vmatrix}$$

$x+\alpha+\beta+\gamma$ $\begin{vmatrix} 1 & \beta & \gamma \\ 1 & x+\beta & \alpha \\ 1 & \beta & x+\gamma \end{vmatrix} = 0$

$R_2 \to R_2 - R$, $R_3 \to R_3 - R$

$\begin{vmatrix} 1 & \beta & \gamma \\ 0 & x & \alpha-\gamma \\ 0 & 0 & x \end{vmatrix} = 0$

$x x = 0$

$x = 0$

If $\begin{vmatrix} 3^2+k & 4^2 & 3^2+3+k \\ 4^2+k & 5^2 & 4^2+4+k \\ 5^2+k & 6^2 & 5^2+5+k \end{vmatrix} = 0$, then the value of k is

(a) 0 (b) −1

(c) 2 (d) 1

$\Rightarrow \begin{vmatrix} 3^2+k & 4^2 & 3 \\ 4^2+k & 5^2 & 4 \\ 5^2+k & 6^2 & 5 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 9+k & 16 & 3 \\ 16+k & 25 & 4 \\ 25+k & 36 & 5 \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 9+k & 16 & 3 \\ 7 & 9 & 1 \\ 16 & 20 & 2 \end{vmatrix}$

$\Rightarrow (9+k)(18-20) - 16(14-16) + 3(140-144)$

$\Rightarrow -18-2k+32-12$

$= -2k+2 = 0$

$k=1$

If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x equals

(a) 2

(b) $-\frac{1}{2}$

(c) 1

(d) $\frac{1}{2}$

$\boxed{A \cdot A^{-1} = I}$

$\Rightarrow \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+0 & 0+0 \\ x+(-x) & 0+2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$2x = 1 \Rightarrow x = 1/2$

$2x = 1$

$x = 1/2$

Column -I		Column- II
A. If $A = [a_{ij}]_{2\times 2}$ is a matrix, where $a_{ij} = \dfrac{(i+j)^2}{2}$, then a_{21} is	1.	$\dfrac{49}{2}$
B. If $B = [b_{ij}]_{2\times 3}$ is a matrix, where $b_{ij} = \dfrac{(i+2j)^2}{2}$, then b_{13} is	2.	12
C. If $C = [c_{ij}]_{3\times 4}$ is a matrix, where $c_{ij} = \dfrac{1}{2}\lvert -3i+j \rvert$, then c_{11} is	3.	2
D. If $D = [d_{ij}]_{3\times 4}$ is a matrix, where $d_{ij} = 2i - j$, then d_{34} is	4.	$\dfrac{9}{2}$

$\dfrac{(1+2(3))^2}{2} = \dfrac{49}{2}$

$a_{21} \Rightarrow i = 2,\ j = 1$

$a_{21} = \dfrac{(2+1)^2}{2} = \dfrac{9}{2}$

Codes

	A	B	C	D
(a)	1	4	2	3
(b)	2	4	3	1
(c)	4	2	1	3
(d)	4	1	2	3

1	2	3	4	5	6	7	8	9	10
B	A	D	A	B	C	A	D	D	D