

The differential equation satisfied by the function

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots \infty}}} \text{ is}$$

(a) $(2y - 1) \frac{dy}{dx} - \sin x = 0$

(b) $(2y - 1) \cos x + \frac{dy}{dx} = 0$

(c) $(2y - 1) \cos x - \frac{dy}{dx} = 0$

(d) $(2y - 1) \frac{dy}{dx} - \cos x = 0$

$$y = \sqrt{\sin x + y}$$

$$y^2 = \sin x + y \rightarrow \text{Diff.}$$

$$2y \frac{dy}{dx} = \cos x + \frac{dy}{dx}$$

$$\frac{dy}{dx} (2y - 1) = \cos x$$



$$\text{Let } f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & , x < 0 \\ c & , x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & , x > 0 \end{cases}$$

If $f(x)$ is continuous at $x=0$, then

- (a) $a+c=0, b=1$
- (b) $a+c=1, b \in \mathbb{R}$
- (c) $a+c=-1, b \in \mathbb{R}$
- (d) $a+c=-1, b=-1$

$\Rightarrow \text{LHL} = \text{RHL} = f(0)$
 $a+2 = \frac{1}{2} = c \Rightarrow a = \frac{1}{2} - 2 = -\frac{3}{2}$

$\text{LHL} = \text{RHL} = f(0)$

$x < 0$

$\text{LHL} = \lim_{x \rightarrow 0^-} \frac{\sin(a+1)x + \sin x}{x} = \frac{0}{0}$

$x = 0$

$x > 0$

LH-Rule $\lim_{x \rightarrow 0^+} \frac{\cos(a+1)x \cdot (a+1) + \cos x}{1}$

$\rightarrow 1 \cdot x(a+1) + 1 = (a+2) = \text{LHL}$

$\neq \text{RHL} \rightarrow \lim_{x \rightarrow 0^+} \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} = \frac{\sqrt{x}(\sqrt{1+bx} - 1)}{bx^{3/2}}$

$\text{RHL} = \frac{\sqrt{1+bx}(-1)}{bx} = \frac{0}{0} (\infty)$

LH-Rule: $\lim_{x \rightarrow 0^+} \frac{1}{2\sqrt{1+bx}} \times b \Rightarrow \frac{1}{2\sqrt{1}} = \frac{1}{2}$

If $y = \sqrt{\frac{1 + \cos 2\theta}{1 - \cos 2\theta}}$, then $\frac{dy}{d\theta}$ at $\theta = \frac{3\pi}{4}$ is :

- (a) -2
- (b) 2
- (c) ± 2
- (d) None of these

$\frac{\frac{\pi}{2} + \frac{\pi}{4}}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{3\pi/4}{\pi/4} = \frac{\pi - \pi/4}{\pi/4}$
 $= -\operatorname{cosec}^2(\pi/4)$
 $= -\operatorname{cosec}^2(\pi/4)$
 $= -\operatorname{cosec}^2(2)$
 $= (-\sqrt{2})^2 = 2 \checkmark$

Solⁿ: $y = \sqrt{\frac{1 + 2\cos^2\theta - 1}{1 - (1 - \sin^2\theta)}} = \sqrt{\frac{2\cos^2\theta}{\sin^2\theta}}$

$y = \cot\theta \rightarrow \frac{dy}{d\theta} = -\operatorname{cosec}^2\theta = -\operatorname{cosec}^2\left(\frac{3\pi}{4}\right)$

$\frac{d}{dx} \left[\sin^{-1} \left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2} \right) \right]$ is equal to

(a) $\frac{1}{2\sqrt{x(1-x)}} - \frac{1}{\sqrt{1-x^2}}$

(b) $\frac{1}{\sqrt{1 - \left\{ x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2} \right\}^2}}$

(c) $\frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{x(1-x)}}$

(d) $\frac{1}{\sqrt{x(1-x)(1-x)^2}}$

$\Rightarrow \sin^{-1} \left[\underbrace{x}_{\sin^2 \alpha} - \underbrace{\sqrt{x}\sqrt{1-x^2}}_{\sin \beta \cos \alpha} \right]$
 $\Rightarrow \sin^{-1} \left[\sin^2 \alpha - \sin \beta \cos \alpha \right]$
 $\Rightarrow \sin^{-1} \left[\sin \alpha \cos \beta - \sin \beta \cos \alpha \right]$
 $\Rightarrow \sin^{-1} \left[\sin(\alpha - \beta) \right] \Rightarrow \frac{d(\alpha - \beta)}{d\alpha}$
 $\Rightarrow \frac{d}{d\alpha} \left[\sin^{-1} x - \sin^{-1} (\sqrt{x}) \right]$
 $\Rightarrow \left[\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} \right] \cancel{\sqrt{x}}$

Rolle's Theorem holds for the function

$x^3 + bx^2 + cx$, $1 \leq x \leq 2$ at the point

$\frac{4}{3}$, the value of b and c are

- (a) $b = 8, c = -5$
- (b) $b = -5, c = 8$
- (c) $b = 5, c = -8$
- (d) $b = -5, c = -8$

If $y = e^{x+e^{x+e^{x+\dots\text{to } \infty}}$, then $\frac{dy}{dx} =$

$y = e^{x+y}$

(a) $\frac{y^2}{1-y}$

(b) $\frac{y^2}{y-1}$

(c) $\frac{y}{1-y}$

(d) $\frac{-y}{1-y}$

$\log y = (x+y) \log e$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{dx}{dx} + \frac{dy}{dx}$

$\frac{dy}{dx} \left(\frac{1}{y} - 1 \right) = 1 \Rightarrow \frac{dy}{dx} = \frac{1 \times y}{1-y}$

Column - I

A. $x = 2at^2, y = at^4$

B. $x = a \cos \theta, y = b \cos \theta$

C. $x = \sin t, y = \cos 2t$

D. $x = 4t, y = \frac{4}{t}$

Column - II

1. $\frac{dy}{dx} = -\frac{1}{t^2}$

2. $\frac{dy}{dx} = t^2$

3. $\frac{dy}{dx} = \frac{b}{a}$

4. $\frac{dy}{dx} = -4 \sin t$

E. $x = \cos \theta - \cos 2\theta,$
 $y = \sin \theta - \sin 2\theta$

5. $\frac{dy}{dx} = \frac{\cos \theta - 2 \cos 2\theta}{2 \sin 2\theta - \sin \theta}$

Codes

	A	B	C	D	E
(a)	2	3	4	1	5
(b)	1	2	4	3	5
(c)	3	1	4	2	5
(d)	4	5	1	2	3

Let $f(x) = \sin x$, $g(x) = x^2$ and $h(x) = \log_e x$.

If $F(x) = (\text{hogof})(x)$, then $F''(x)$ is equal to

- (a) $\text{a cosec}^3 x$
- (b) $2 \cot x^2 - 4x^2 \text{ cosec}^2 x^2$
- (c) $2x \cot x^2$
- (d) $-2 \text{ cosec}^2 x$

$\frac{d}{dx} \left[\log \left\{ e^x \left(\frac{x-2}{x+2} \right) \right\}^{3/4} \right]$ is equal to

(a) 1

(b) $\frac{x^2 + 1}{x^2 - 4}$

(c) $\frac{x^2 - 1}{x^2 - 4}$

(d) $e^x \frac{x^2 - 1}{x^2 - 4}$

1	2	3	4	5	6	7	8	9	10
D	C	A	C	B	C	B	A	D	C