AOD

Ex:- (1) a balloon, which remains $\boxed{\text{spherical}}$ has variable dia $\frac{3}{2}(2x+1)$. find rate of change of vol. wrt x.

Ex:- (2) a particle moves along the curve $6y = x^3 + 2$. find the points on the curve at which y-cordinate is changing 8 times as fast as x-cordinate.

Ex-(1) Soln:-

dia $= \frac{3}{2}(2x+1)$ find:- $\left[\frac{dV}{dx} = ?\right]$

$r = \frac{3}{4}(2x+1)$

$\because$ Volu. of sphere $\Rightarrow V = \frac{4}{3}\pi r^3$

$\Rightarrow V = \frac{4}{3}\pi\left[\frac{3}{4}(2x+1)\right]^3$

$\Rightarrow \left[V = \frac{\cancel{4}}{\cancel{3}}\pi \times \frac{\cancel{27}\,9}{\cancel{64}\,16}(2x+1)^3\right]$

$\Rightarrow \frac{d}{dx}\left[\frac{9}{16}\pi(2x+1)^3\right] = \frac{9}{\cancel{16}\,8}\pi \times 3(2x+1)^2 \times \cancel{2}$

$\Rightarrow \frac{27\pi}{8}(2x+1)^2$ Ans

AOD

Ex:- (1) a balloon, which remains $\boxed{\text{spherical}}$ has variable dia $\frac{3}{2}(2x+1)$.
find rate of change of vol. wrt x.

Ex:- (2) a particle moves along the curve $6y = x^3 + 2$. find the points on the curve at which $\boxed{\text{y-Cordinate is Changing}}$ 8 times as fast as x-Cordinate.

Sol^n:- $\because \left[6y = x^3 + 2\right]$ given:- $\left[\frac{dy}{dt} = 8 \times \frac{dx}{dt}\right] \Rightarrow \left[\frac{dy_1}{dt} = 8 \times \frac{dx_1}{dt}\right]$ — (1)

$\Rightarrow$ let the points is (x_1, y_1) on the curve.

$\Rightarrow$ then the curve become:- $\left[6y_1 = x_1^3 + 2\right]$ — (11)

so Now diff eq^n (1) wrt t:- $\left[6 \cdot \frac{dy_1}{dt} = 3 \cdot x_1^2 \times \frac{dx_1}{dt}\right]$ — (3)

$\Rightarrow$ from eq^n (1) & (3):- $\cancel{2}\ \cancel{6} \cdot 8 \cdot \frac{dx_1}{dt} = \cancel{3} \cdot x_1^2 \times \frac{dx_1}{dt} \Rightarrow x_1^2 = 16 \Rightarrow x_1 = \pm 4$

put $x = \pm 4$ in eq^n (2):- $6y_1 = 66$, $y_1 = 11$ Ans | $6y_1 = -62$, $y_1 = -\frac{31}{3}$ Ans

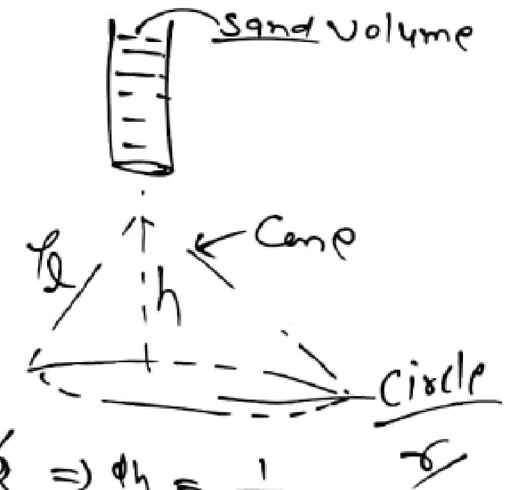
So the points are:- $(4, 11)$ & $(-4, -31/3)$ Ay

AOD

ex:- Sand is pouring from a pipe at a rate of $12\,cm^3/sec$. the falling sand forms a cone on the the ground such a way that the height of the cone is always one-sixth of the radius of the base. How fast is heigth of the sand cone increasing when height 4 cm.

Sol^n:- given: $\frac{dV}{dt} = 12\,cm^3/sec.$

$$h = \frac{1}{6} \times r \Rightarrow \boxed{r = 6h}$$



find:- $\left[\left.\frac{dh}{dt}\right|_{h=4} = ?\right]$

$$\because \frac{dV}{dt} = 12 \Rightarrow \frac{d}{dt}\left[\frac{1}{3}\pi r^2 h\right] = 12 \Rightarrow \frac{1}{3}\pi\left(\frac{d}{dt}(6h)^2 \times h\right) = 12$$

$$\Rightarrow \frac{1}{\cancel{3}}\pi \times \cancel{36}^{12} \times \frac{d}{dt}(h^3) = 12 \Rightarrow \cancel{12}\pi \times 3h^2 \times \frac{dh}{dt} = \cancel{12} \Rightarrow \frac{dh}{dt} = \frac{1}{\pi \cdot 3h^2}$$

$$\Rightarrow \left.\frac{dh}{dt}\right|_{h=4} = \frac{1}{\pi \cdot 3 \cdot (4)^2} = \frac{1}{48\pi}\ cm/sec.\ \text{Ans}$$

AOD

Ex1 - Tangent of an angle increases four times as the angle itself. at what rate the sine of the angle increases w.r.t the angle.

Sol^n:- $\because$ given:- $\left[\frac{d(\tan\theta)}{dt} = 4\times\left(\frac{d\theta}{dt}\right)\right]$

$$\Rightarrow \frac{\frac{d.\tan\theta}{\cancel{dt}}}{\frac{d\theta}{\cancel{dt}}} = 4 \Rightarrow \frac{d(\tan\theta)}{d\theta} \Rightarrow \sec^2\theta = 4$$

$$\Rightarrow \boxed{\sec\theta = \pm 2}$$

find:- $\frac{d(\sin\theta)}{d\theta} = ? \Rightarrow \because \frac{d(\sin\theta)}{dt} = \boxed{\cos\theta = \pm\frac{1}{2}}$

$\cos\theta = +\frac{1}{2}$ Ans

AOD

Ex:- Tangent of an angle increases four times as the angle itself. at what rate the sine of the angle increases w.r.t the angle.

Solⁿ:- ∵ given:- $\dfrac{d(\tan\theta)}{dt} = 4\times\dfrac{d\theta}{dt}$

Ex:- a spherical balloon is inflated at rate of 35 cc/min. (cubic cm) the rate of increase of S.A. (Surface Area) of the balloon when Dia is 14 cm?

Ex:- The length x of Rectangle is ↓ at 5 cm/min & width y is ↑ at 4 cm/min. when x = 8 cm & y = 6 cm find.

rate of change i) perimeter
ii) Area.