Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}.

(a) R is reflexive and symmetric but not transitive φ

(b) R is reflexive and transitive but not symmetric

(c) R is symmetric and transitive but not reflexive φ

(d) R is equivalence relation φ

(1, 2) (1, 3) (3, 2) φ φ

$\downarrow$

R

a, b ∈ R b, c ∈ R

$\Downarrow$

a, c ∈ R

Let $f(x) = \dfrac{ax+b}{cx+d}$. Then $fof(x) = x$ provided that

(a) $d = -a$ (b) $d = a$

(c) $a = b = c = d = 1$ (d) $a = b = 1$

LHS → RHS

$\boxed{d = -a}$ $d^2 = a^2$

$f(x)$

$\Rightarrow f(f(x)) = x \Rightarrow f\left(\dfrac{ax+b}{cx+d}\right) = x$

$$\Rightarrow \frac{a\left[\dfrac{ax+b}{cx+d}\right] + b}{c\left[\dfrac{ax+b}{cx+d}\right] + d} = x$$

$$\Rightarrow \frac{a^2x + ab + bcx + bd}{acx + cb + cdx + d^2} = x$$

$$\Rightarrow \frac{x[a^2+bc] + [ab+bd]}{x[ac+cd] + [cb+d^2]} = x$$

Solve

$$\frac{x[a^2+bc] + \cancel{ab} - \cancel{ab}}{x[\cancel{ac} + (-\cancel{ac})] + cb + a^2} = x$$

$$\frac{x[a^2+bc]}{cb+a^2} = x$$

$\boxed{x = x}$

$\boxed{fof(x) = x}$

Let f : N → R be the function defined by $f(x)=\frac{2x-1}{2}$ and g : Q → R be another function defined by g(x) = x+2. Then (gof) $\frac{3}{2}$ is

(a) 1 (b) 0

(c) $\frac{7}{2}$ (d) 3

Sol^n: $gof(x) \rightarrow g[f(x)] = g\left[\frac{2x-1}{2}\right]$ → 3/2

$\Rightarrow \frac{2x-1}{2}+2 = \frac{2x-1+4}{2} = \frac{2x+3}{2}$

$\Rightarrow gof(3/2) = \frac{\cancel{2}\times\frac{3}{\cancel{2}}+3}{2} = \frac{6}{2} = 3$

ABLES KOTA

If the binary operation * is defined on the set Q^+ of all positive rational numbers by $\boxed{a * b = \frac{ab}{4}}$. Then $3 * \left(\frac{1}{5} * \frac{1}{2}\right)$ is equal to

(a) $\frac{3}{160}$

(b) $\frac{5}{160}$

(c) $\frac{3}{10}$

(d) $\frac{3}{40}$

$$\frac{\frac{1}{5} \times \frac{1}{2}}{4}$$

$$3 \times \frac{1}{40} = \frac{3 \times \frac{1}{40}}{4}$$

$$\frac{3}{40} \times \frac{1}{4} = \frac{3}{160}$$

The inverse of the function

$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 2$ is — y → given

inverse → $x = y$

(a) $\log_e\left(\frac{x-3}{x-1}\right)^{1/2}$ (b) $\log_e\left(\frac{x-1}{3-x}\right)^{1/2}$

(c) $\log_e\left(\frac{x+2}{x-3}\right)^{1/2}$ (d) $\log_e\left(\frac{x+1}{x-2}\right)^{1/2}$

Sol: $y - 2 = \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} \Rightarrow y - 2 = \frac{e^{2x} - 1}{e^{2x} + 1}$

$\Rightarrow y \cdot e^{2x} + y - 2 \cdot e^{2x} - 2 = e^{2x} - 1$

$\Rightarrow y \cdot e^{2x} - 2e^{2x} - e^{2x} = -1 + 2 - y$

$\Rightarrow ye^{2x} - 3e^{2x} = 1 - y$

$\rightarrow e^{2x}(y - 3) = 1 - y$

$\rightarrow \boxed{e^{2x} = \frac{1-y}{y-3}}$

$\rightarrow \log e^{2x} = \log\left(\frac{1-y}{y-3}\right)$

$2x \log e = \log\left(\frac{1-y}{y-3}\right) \Rightarrow x = \frac{1}{2}\log\left(\frac{1-y}{y-3}\right)$

$x = \cancel{\frac{1}{2}}\left[\log_e\left(\frac{1-y}{y-3}\right)\right]^{1/2} \rightarrow$ inverse

So: inverse of $y \Rightarrow \log_e\left(\frac{1-x}{x-3}\right)^{1/2}$

$\log_e\left(\frac{x-1}{3-x}\right)^{1/2}$ Ans

If $f : R \to R$ be given by $f(x) = \left(3 - x^3\right)^{\frac{1}{3}}$,

then fof (x) is

(a) $x^{\frac{1}{3}}$ $fof(x) = ?$ (b) x^3

(c) x $\Rightarrow f[f(x)]$ (d) $(3 - x^3)$

$\Rightarrow f\left[(3-x^3)^{1/3}\right] \to \left[3 - \left\{(3-x^3)^{1/3}\right\}^3\right]^{1/3}$

$f(x)$

$(3 - 3 + x^3)^{1/3}$

$(x^3)^{1/3} = x$

The value of x obtained from the equation

$$\begin{vmatrix} x+\alpha & \beta & \gamma \\ \gamma & x+\beta & \alpha \\ \alpha & \beta & x+\gamma \end{vmatrix} = 0 \text{ will be}$$

(a) 0 and $-(\alpha+\beta+\gamma)$

(b) 0 and $\alpha+\beta+\gamma$

(c) 1 and $(\alpha-\beta-\gamma)$

(d) 0 and $\alpha^2+\beta^2+\gamma^2$

$x = -(\alpha+\beta+\gamma)$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} x+\alpha+\beta+\gamma & \beta & \gamma \\ x+\alpha+\beta+\gamma & x+\beta & \alpha \\ x+\alpha+\beta+\gamma & \beta & x+\gamma \end{vmatrix}$$

$(x+\alpha+\beta+\gamma)$ $\begin{vmatrix} 1 & \beta & \gamma \\ 1 & x+\beta & \alpha \\ 1 & \beta & x+\gamma \end{vmatrix} = 0$

$R_2 \to R_2 - R$, $R_3 \to R_3 - R$

$$\begin{vmatrix} 1 & \beta & \gamma \\ 0 & x & \alpha-\gamma \\ 0 & 0 & x \end{vmatrix} = 0$$

$x x = 0$

$x = 0$

If $\begin{vmatrix} 3^2+k & 4^2 & 3^2+3+k \\ 4^2+k & 5^2 & 4^2+4+k \\ 5^2+k & 6^2 & 5^2+5+k \end{vmatrix} = 0$, then the value of k is

(a) 0
(b) -1
(c) 2
(d) 1

$\Rightarrow \begin{vmatrix} 3^2+k & 4^2 & 3 \\ 4^2+k & 5^2 & 4 \\ 5^2+k & 6^2 & 5 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 9+k & 16 & 3 \\ 16+k & 25 & 4 \\ 25+k & 36 & 5 \end{vmatrix}$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$\Rightarrow \begin{vmatrix} 9+k & 16 & 3 \\ 7 & 9 & 1 \\ 16 & 20 & 2 \end{vmatrix}$

$\Rightarrow 9+k(18-20) - 16(14-16) + 3(140-144)$

$\Rightarrow -18-2k+32-12$

$= -2k+2 = 0$

$k=1$

Ans

If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$,

then x equals

(a) 2 (b) $-\frac{1}{2}$

(c) 1 (d) $\frac{1}{2}$

$$\boxed{A \cdot A^{-1} = I}$$

$$\Rightarrow \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+0 & 0+0 \\ x+(-x) & 0+2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2x = 1 \Rightarrow x = 1/2$ $2x = 1$

$x = 1/2$

Column -I	Column- II
A. If $A=[a_{ij}]_{2\times 2}$ is a matrix, where $a_{ij}=\frac{(i+j)^2}{2}$, then a_{21} is	1. $\frac{49}{2}$
B. If $B=[b_{ij}]_{2\times 3}$ is a matrix, where $b_{ij}=\frac{(i+2j)^2}{2}$, then b_{13} is	2. 1 2
C. If $C=[c_{ij}]_{3\times 4}$ is a matrix, where $c_{ij}=\frac{1}{2}\lvert -3i+j\rvert$, then c_{11} is	3. 2
D. If $D=[d_{ij}]_{3\times 4}$ is a matrix, where $d_{ij}=2i-j$, then d_{34} is	4. $\frac{9}{2}$

$\frac{(1+2(3))^2}{2} = \frac{49}{2}$

$a_{21} \Rightarrow i=2,\ j=1$

$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$

Codes

	A	B	C	D
(a)	1	4	2	3
(b)	2	4	3	1
(c)	4	2	1	3
(d)	4	1	2	3

1	2	3	4	5	6	7	8	9	10
B	A	D	A	B	C	A	D	D	D