Conti. & Diff

Ex:- $y = \sqrt{(a-x)(x-b)} - (a-b)\tan^{-1}\sqrt{\frac{a-x}{x-b}}$ then $\frac{dy}{dx}$

Soln: let $x = a\cos^2\theta + b\sin^2\theta$

So: $a - x = a - (a\cos^2\theta + b\sin^2\theta)$

$(a-x) = a - a\cos^2\theta - b\sin^2\theta$

$= a(1-\cos^2\theta) - b\sin^2\theta$

$= a\sin^2\theta - b\sin^2\theta$

$(a-x) = (a-b)\sin^2\theta$ — (1)

$\Rightarrow (x-b) = a\cos^2\theta + b\sin^2\theta - b$

$= a\cos^2\theta - b(1-\sin^2\theta)$

$x - b = (a-b)\cos^2\theta$ — (2)

$\Rightarrow \frac{a-x}{x-b} = \frac{(a-b)\cdot\sin^2\theta}{(a-b)\cdot\cos^2\theta} = \tan^2\theta$ — (3)

$\Rightarrow$ put eqn (1), (2) & (3) in y.

$y = \sqrt{(a-b)^2\sin^2\theta\cdot\cos^2\theta} - (a-b)\tan^{-1}\sqrt{\tan^2\theta}$

$y = (a-b)\cdot\sin\theta\cdot\cos\theta - (a-b)\cdot\tan^{-1}\cdot\tan\theta$

$y = (a-b)\cdot\left[\frac{2\sin\theta\cdot\cos\theta}{2} - \theta\right]$

$y = (a-b)\cdot\left[\frac{\sin 2\theta}{2} - \theta\right]$

$\frac{dy}{d\theta} = (a-b)\left[\frac{1}{2}\cos 2\theta \times 2 - 1\right] = (a-b)(\cos 2\theta - 1)$ — (4)

Now $\frac{dx}{d\theta} = a\cdot 2\cos\theta\times(-\sin\theta) + b\cdot 2\sin\theta\cdot\cos\theta$

$\frac{dx}{d\theta} = -a\sin 2\theta + b\sin 2\theta = \sin 2\theta\,(b-a)$ — (5)

So $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-(b-a)(\cos 2\theta - 1)}{(b-a)\cdot\sin 2\theta} = \frac{1-\cos 2\theta}{\sin 2\theta}$ Ans $= \frac{1 - 1 + 2\sin^2\theta}{2\sin\theta\cdot\cos\theta}$

$\frac{dy}{dx} = \frac{2\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$ Ans

from eqn (3) $\Rightarrow \tan\theta = \sqrt{\frac{a-x}{x-b}} = \frac{dy}{dx}$ Ans

Conti. & Diff

Q. $y = x^{x^{x \cdots \infty}}$

$\Rightarrow \boxed{y = x^{y}}$

$\Rightarrow \left[\log y = y \cdot \log x\right]$

$\frac{1}{y} \cdot \frac{dy}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$

$\frac{dy}{dx}\left(\frac{1}{y} - \log x\right) = \frac{y}{x}$

$\frac{dy}{dx} = \frac{y}{x}\left[\frac{y}{1 - y \cdot \log x}\right] = \frac{y^2}{x(1 - y \cdot \log x)}$ Ans

Q. if $[z = x \cdot e^{y}]$

$x = t$, $y = 1 + 3t$

$\frac{dz}{dt}$

$\Rightarrow \left[z = \underset{u}{t} \cdot \underset{v}{e^{(1+3t)}}\right] \rightarrow$ Diff wrt t

$\Rightarrow \frac{dz}{dt} = t \cdot e^{(1+3t)} \times (0 + 3) + e^{(1+3t)} \cdot 1$

$\frac{dz}{dt} = 3t \cdot e^{(1+3t)} + e^{(1+3t)}$

$= e^{(1+3t)}[3t + 1]$ Ans

$= e^{y}[y]$ Ans

Conti. & Diff. # $y^2 \cdot p'''(x)$

Ex:- if $y^2 = p(x)$ is polynomial of degree 3. then → $2\dfrac{d}{dx}\left[y^3 \dfrac{d^2y}{dx^2}\right] = ?$

Sol^n:- $y^2 = p(x)$

→ Diff → $2 \cdot y \cdot y' = p'(x) \Rightarrow y' = \dfrac{p'(x)}{2y}$

→ Diff again → $2\left[y \cdot y'' + y' \cdot y'\right] = p''(x)$

$\Rightarrow 2 \cdot y \cdot y'' + 2(y')^2 = p''(x)$

$\Rightarrow y'' = \dfrac{p''(x) - 2(y')^2}{2y}$

$\Rightarrow y'' = \dfrac{p''(x)}{2y} - \dfrac{(y')^2}{y} = \dfrac{d^2y}{dx^2}$

Multiply with y^3

$\Rightarrow y^3 \cdot y'' = \dfrac{y^3 \cdot p''(x)}{2y} - \dfrac{y^3 \cdot (y')^2}{y}$

→ $y^3 \cdot y'' = \dfrac{y^2 \cdot p''(x)}{2} - y^2 \cdot (y')^2$

→ diff again wrt → x

$\Rightarrow \dfrac{d}{dx}\left[y^3 \cdot y''\right] = \dfrac{d}{dx}\left\{\dfrac{y^2}{2} \cdot p''(x) - y^2 \cdot \left(\dfrac{p'(x)}{2y}\right)^2\right\}$

$\Rightarrow \dfrac{d}{dx}\left[y^3 \cdot y''\right] = \dfrac{d}{dx}\left[\dfrac{y^2}{2} \cdot p''(x) - \dfrac{(p'(x))^2}{4}\right]$

$\Rightarrow \dfrac{d}{dx}\left[y^3 \cdot y''\right] = \dfrac{1}{2}\left[y^2 \cdot p'''(x) + p''(x) \cdot 2y \cdot y'\right] - \dfrac{1}{4} \cdot 2\left[p'(x)\right] \times p''(x)$

$= 2 \cdot \dfrac{d}{dx}\left(y^3 \cdot y''\right) = y^2 \cdot p'''(x) + p''(x) \cdot p'(x) - p'(x) \cdot p''(x)$