# # Conti. & Diff. #

Ex:- given $f(x) = \frac{1}{1-x}$ find point of discontinuity of the composite fun. $y = f[f\{f(x)\}]$

Here $f(x)$ is disconti at $x = 1$ &
$f\{f(x)\}$ ——— " at $x = 0$
& $f[f\{f(x)\}]$ conti at every where
$\therefore$ $y$ is discont at $x = 0$ & $x = 1$

Sol$^n$:- $\because$ $\boxed{f(x) = \frac{1}{1-x}}$ — ①

$\therefore \Rightarrow f\{f(x)\} = \frac{1}{1-f(x)} = \frac{1}{1-\frac{1}{1-x}} = \frac{1}{\frac{\cancel{1}-x-\cancel{1}}{1-x}} = \frac{1-x}{-x} = \frac{x-1}{x}$

$\Rightarrow \boxed{f\{f(x)\} = \frac{x-1}{x}}$ — ②

again:- $f[f\{f(x)\}] = \frac{f(x)-1}{f(x)} = \frac{\frac{1}{1-x}-1}{\frac{1}{1-x}} = \frac{\frac{\cancel{1}-\cancel{1}+x}{\cancel{1-x}}}{\frac{1}{\cancel{1-x}}} = x$

$\Rightarrow$ $\boxed{\text{So } y = f[f\{f(x)\}] = x}$ → point of disconti

# Conti. & Diff.

Ex: $2y = \left[\cot^{-1}\left(\frac{\sqrt{3}\cos x + \sin x}{\cos x - \sqrt{3}\sin x}\right)\right]^2$

$x \in (0, \pi/2)$ then $dy/dx = ?$

Sol$^n$:- Divide & Mutiply by $\cos x$.

$\Rightarrow 2y = \left[\cot^{-1}\left[\frac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x}\right]\right]^2$

$\Rightarrow 2y = \left[\cot^{-1}\left[\frac{\tan\frac{\pi}{3} + \tan x}{1 - \tan\frac{\pi}{3}\cdot\tan x}\right]\right]^2$

$\Rightarrow 2y = \left[\cot^{-1}\left\{\underbrace{\tan\left(\frac{\pi}{3} + x\right)}_{\theta}\right\}\right]^2$

$\because \cot^{-1}\theta + \tan^{-1}\theta = \pi/2$

$\cot^{-1}\theta = \pi/2 - \tan^{-1}\theta$

$\Rightarrow 2y = \left[\frac{\pi}{2} - \cancel{\tan^{-1}}\,\cancel{\tan}\left(\frac{\pi}{3} + x\right)\right]^2$

$\Rightarrow 2y = \left[\frac{\pi}{2} - \frac{\pi}{3} - x\right]^2 = \left[\pi/6 - x\right]^2$

$\rightarrow$ Diff wrt $x$

$\cancel{2}\cdot\frac{dy}{dx} = \cancel{2}\left[\frac{\pi}{6} - x\right] \times (-1)$

$\frac{dy}{dx} = x - \pi/6$

# Conti. & Diff.

Ex:- $f(1)=1$, $f'(1)=3$ then Derivative of

$f[f\{f(x)\}]+[f(x)^2]$ is = ?

$\cos^2\theta = \cos(\theta)^2$

Soln:- let $y = f[f\{f(x)\}]+[f(x)^2]$

→ Diff → $y' = f'[f\{f(x)\}] \times f'\{f(x)\} \times f'(x) + 2\cdot f(x) \times f'(x)$

⇒ put $x=1$ ⇒ $y' = f'[f\{f(1)\}] \times f'\{f(1)\} \times f'(1) + 2\cdot f(1) \times f'(1)$

⇒ $y' = f'[f(1)] \times f'(1) \times 3 + 2 \times 1 \times 3$

→ $y' = f'(1) \times 3 \times 3 + 6$

$y' = 3 \times 3 \times 3 + 6 = 33$ Ans

# Conti. & Diff.

**Ex:-** if $f(x) = \begin{cases} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+4} - 2}, & x \neq 0 \\ a, & x = 0 \end{cases}$

then the value of $a$, for which $f(x)$ may be Conti at $x = 0$.

$\because$ Limit $= f(0)$

$\boxed{-8 = a}$ Ans

**Soln:** $\because$ $x = 0$ Conti $\rightarrow$ $\therefore$ $f(0) = \boxed{\lim_{x \to 0} f(x)}$

$\because \lim_{x\to 0} f(x) \Rightarrow \text{RHL} \Rightarrow \lim_{x \to 0^+} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+4} - 2} = \dfrac{\cos 2x - 1}{\sqrt{x^2+4} - 2} = \dfrac{0}{0}$ (undefine)

$\rightarrow$ L.H. Rule: $\lim_{x \to 0^+} \left[ \dfrac{-2 \cdot \sin 2x - 0}{\dfrac{1}{2\sqrt{x^2+4}} \times 2x - 0} \right] \Rightarrow \left[ \lim_{x \to 0^+} \dfrac{-2 \sin 2x \sqrt{x^2+4}}{x} \right] = \dfrac{0}{0}$ (undefine)

$\Rightarrow$ L.H. Rule:- $\lim_{x \to 0^+} -2 \left[ \dfrac{\sin 2x \cdot \dfrac{1}{2\sqrt{x^2+4}} \times 2x + \sqrt{x^2+4} \cdot \cos 2x \times 2}{1} \right] \Rightarrow -2 [0 + 2 \times 1 \times 2]$

$= -8$ Ans