Conti & Diff

Ex:- find derivative of $\frac{1}{x^5-9}$ wrt x^2

Sol^n:-

let $f(x) = \frac{1}{x^5-9} = (x^5-9)^{-1}$

diff wrt → x ⇒ $f'(x) = \frac{-1}{(x^5-9)^2} \times 5x^4$

$$f'(x) = \frac{-5x^4}{(x^5-9)^2}$$

$g(x) = x^2$

⇒ $g'(x) = 2x$

Sol- derivative of $\frac{1}{x^5-9}$ wrt → x^2 ⇒ $\frac{f'(x)}{g'(x)} = \frac{-5x^4}{(x^5-9)^2} \times \frac{1}{2x}$

$$= \frac{-5x^3}{2(x^5-9)^2}$$ Ans

Ex:- $f(x) = \begin{cases} [x]+[-x] ; & x \neq 2 \\ \lambda ; & x = 2 \end{cases}$, if $f(x)$ is conti at $x=2$, find value of λ.

Soln:- $\because$ fun. is conti. so:- $LHL = RHL = f(2)$ —— (1)

$\because$ $RHL \Rightarrow \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} [x]+[-x] \Rightarrow \lim_{x \to 2^+} [x+h] + [-(x+h)]$

$RHL = [2+h] + [-(2+h)] = 2 + (-3) = -1$

$\Rightarrow LHL = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} [x]+[-x] \Rightarrow \lim_{x \to 2^-} [(x-h)+[-(x-h)]]$

$= [2-h] + [-(2-h)] = 1 + (-2) = -1$

$\because f(x)$ Conti $\rightarrow$ so (1):- $-1 = -1 = \lambda$ Ans

Ex:- ✪ the value of $f(0)$ so that the func

$$f(x) = \frac{\log(1+x/9) - \log(1-x/6)}{x}$$ is Conti at $x=0$.

Sol^n:- ∵ $f(x)$ is Conti → ∴ $f(0) = LHL = RHL$

∵ $\lim_{x\to 0} f(x) = \left(\frac{0}{0}\right)$ not define

$$\Rightarrow \lim_{x\to 0} \frac{\log(1+x/9)}{x} - \lim_{x\to 0} \frac{\log(1-x/6)}{x}$$

$$\rightarrow \lim_{x\to 0} \frac{1}{x} \times \frac{x}{9} \cdot \frac{\log(1+x/9)}{x/9} - \lim_{x\to 0} \frac{1}{x} \times \frac{-x}{6} \cdot \frac{\log(1-x/6)}{-x/6}$$

$$\Rightarrow \frac{1}{9} - 1\left(\frac{-1}{6}\right) \Rightarrow \frac{1}{9} + \frac{1}{6} = \frac{9+6}{96}$$ Ans

$$\lim_{x\to 0} \frac{\log(1+x)}{x} = \frac{0}{0}$$

L.H. Rule:- $\lim_{x\to 0} \frac{1}{1+x}$

$$\Rightarrow \lim_{x\to 0} \frac{1}{1+x} = \lim_{x\to 0} 1 =$$

$$\Rightarrow \lim_{x\to 0} \frac{\log(1+x)}{x} = 1$$

Ex:- $y = (\tan x)^{(\tan x)^{\tan x}}$ → find $\frac{dy}{dx}$ at $x = \pi/4$.

→ Soln:- take log → $\log y = \log (\tan x)^{(\tan x)^{\tan x}}$ ← $\log M^N$

→ $\log y = (\tan x)^{\tan x} \cdot \log(\tan x)$ $\quad M \cdot N$

→ take log again:- $\log(\log y) = \log\left\{(\tan x)^{\tan x} \cdot \log(\tan x)\right\}$

⇒ $\log(\log y) = \log(\tan x)^{\tan x} + \log \cdot \log(\tan x)$

⇒ $\log(\log y) = \overset{u}{\tan x} \cdot \overset{v}{\log}(\tan x) + \log\{\log(\tan x)\}$

diff ⇒ $\frac{1}{\log y} \times \frac{1}{y} \times \frac{dy}{dx} = \left[\tan x \cdot \frac{1}{\tan x} \times \sec^2 x + \log(\tan x) \cdot \sec^2 x\right] + \frac{1}{\log(\tan x)} \times \frac{1 \times \sec^2 x}{\tan x}$

$$\frac{dy}{dx} = (\log y)\, y \left[\left\{ \sec^2 x \,(1 + \log(\tan x)) \right\} + \frac{\sec^2 x}{\tan x \cdot \log(\tan x)} \right]$$

$x = \pi/4$

$\because \quad y = (\tan x)^{(\tan x)}$

y at $x = \frac{\pi}{4} \Rightarrow y = \left(\tan \frac{\pi}{4}\right)^{(\tan \pi/4)} = (1)^{(1)} = 1$

$\Rightarrow y = 1$

take log $\rightarrow \log y = \log 1 = 0$

So: $\frac{dy}{dx} = 0 \times [\quad] = 0$