Conti & Diff.

1) **Rolle's Theorem** :- if a function $f(x)$ is

a) $f(x)$ is Conti. in $[a, b]$

b) $f(x)$ is diff. in (a, b)

c) $f(a) = f(b)$

Then there exist a point c such that :- $\boxed{f'(c) = 0}$

for $\boxed{c \in (a, b)}$

$[2, 5]$

$f'(x) = 0$

Ex:- Verify Rolle's Theorem:-

$f(x) = x^3 - 3x^2 + 2x$ for $x \in [0, 2]$

Sol^n:- ∵ $f(x) = x^3 - 3x^2 + 2x$ is a polynomial so:-

✓ i) $f(x)$ is Conti in $[0, 2]$

✓ ii) $f(x)$ is diff in $(0, 2)$

✓ iii) $f(a) = f(b)$

∵ $f(a) = 0$, $f(b) = 8 - 12 + 4 = 0$

∴ $f(a) = f(b) = 0$

Thus there exist a point $c \in (a, b)$ such that

$f'(c) = 0$

So! $f'(x) = 3x^2 - 6x + 2 \Rightarrow f'(c) = 3c^2 - 6c + 2$

Now $f'(c) = 0$

$\Rightarrow 3c^2 - 6c + 2 = 0$

$\Rightarrow c = \dfrac{+6 \pm \sqrt{36 - 4 \times 3 \times 2}}{6}$

$\Rightarrow c = \dfrac{6 \pm \sqrt{12}}{6} = \dfrac{6 \pm 2\sqrt{3}}{6}$

$c = \dfrac{\cancel{2}(3 \pm \sqrt{3})}{\cancel{6}\,3}$

$$C = \frac{3 \pm \sqrt{3}}{3}$$

so $C = \frac{3+\sqrt{3}}{3}$, $C = \frac{3-\sqrt{3}}{3}$

$C = \frac{3+1.73}{3}$ $\quad$ $C = \frac{3-1.73}{3}$

$C = \frac{4.73}{3}$ $\quad$ $C = \frac{1.27}{3}$

$\boxed{C = 1.57}$ $\quad$ $\boxed{C = 0.42}$

Here $C = 1.57 \in (0, 2)$

& $C = 0.42 \in (0, 2)$ so Rolle's Theorem verify

Lagrange Mean value Theorem :-

→ if a fuction $f(x)$ is- $\quad f'(x) \to \underline{f'(c)}$

a) $f(x)$ is Conti. in $[a, b]$

b) $f(x)$ is diff. in (a, b)

→ Then there exist a point $C \in (a, b)$ such that

$$\boxed{\underline{f'(c)} = \frac{f(b) - f(a)}{b - a}}$$

$$\Rightarrow \boxed{f(b) = f(a) \;\Rightarrow\; f'(c) = 0 \;\Rightarrow\; c \in (a, b)}$$

Q. verify Lagrange MVT if

$$f(x) = x^3 - 5x^2 - 3x \longrightarrow [\overset{a}{1}, \overset{b}{3}]$$

Solⁿ:- ∵ given f(x) is polynomial.

∴ i) f(x) is Conti in [1, 3]

ii) f(x) is diff. in (1, 3)

then there exist a point $c \in (1, 3)$ such that $\boxed{f'(c) = \dfrac{f(b) - f(a)}{b - a}}$

Sol:- $\boxed{f'(x) = 3x^2 - 10x - 3}$

$f'(c) = 3c^2 - 10c - 3$

$f(b) = f(3) = 27 - 45 - 9 = -27$

$f(a) = f(1) = 1 - 5 - 3 = -7$

So:- $3c^2 - 10c - 3 = \dfrac{-27 - (-7)}{3 - 1}$

$\Rightarrow 3c^2 - 10c - 3 = \dfrac{-20}{2} = -10$

$\Rightarrow \boxed{3c^2 - 10c + 7 = 0}$

$3c^2 - 3c - 7c + 7 = 0$

$3c(c-1) - 7(c-1) = 0$

$\Rightarrow c = 1$ (N.P.) $c = 7/3 = 2.33 \in (1,3)$ M.P.

Ex :- if $f(x) = 8x^2 - 7x + 5$ on the interval $[\overset{a}{-6}, \overset{b}{6}]$

find value of $\underline{c}$ that satisfy Lagrange MVT.

Sol^n :- ∵ Here $f(x)$ satisfy LMVT.

∴ $c \in (a, b)$ such that :- $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

So! $f'(x) = 16x - 7 \Rightarrow f'(c) = 16c - 7$

$f(b) = f(6) = 251$ & $f(a) = 335$

So $16c - 7 = \dfrac{251 - 335}{6 - (-6)} \Rightarrow 16c - \cancel{7} = -\dfrac{84}{12} \cancel{7}$

$= 16c = 0$

$\Rightarrow c = 0$ Ans

$$\begin{array}{r} 335 \\ 251 \\ \hline 84 \end{array}$$

Ex: using Lagrange MVT prove that:

$$\boxed{|\cos a - \cos b| \leq |a - b|}$$

Sol^n:- let $\boxed{f(x) = \cos x}$ in interval $[a, b]$

So:- i) Here $f(x)$ is Conti in $[a, b]$

ii) $f(x)$ is diff in (a, b)

$\therefore$ $f'(c) = \dfrac{f(b) - f(a)}{b - a} \Rightarrow \boxed{-\sin c = \dfrac{\cos b - \cos a}{b - a}}$

take Mod. $\rightarrow$ $|-\sin c| = \dfrac{|\cos b - \cos a|}{|b - a|} \Rightarrow |\sin c| = \dfrac{|\cos a - \cos b|}{|b - a|}$

$1 \geq \dfrac{|\cos a - \cos b|}{|b - a|}$

$\Rightarrow |b - a| \geq |\cos a - \cos b|$

H.P.