Conti & Diff

Ex:- $[y = x^3 \cdot \log x]$ → find $\frac{d^2y}{dx^2} = ?$

(u · v)

→ Diff wrt x

$$\Rightarrow \frac{dy}{dx} = x^3 \cdot \frac{1}{x} + \log x \cdot 3x^2 = x^2 + \log x \cdot 3x^2$$

(u · v)

$$\rightarrow \boxed{\frac{dy}{dx} = x^2[1 + 3\log x]}$$

→ Diff again wrt → x

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = x^2\left[0 + 3\cdot\frac{1}{x}\right] + [1 + 3\log x]\,2x$$

$$\rightarrow \boxed{\frac{d^2y}{dx^2}} = 3x + 2x + 3 \cdot 2x \log x$$

$$= [5x + 6x \log x]$$ Ans

H.W → d^2y/dx^2

Ex:-1) $x^2 + 3x + 2$

2) $y = \log(\log x)$

3) $y = e^{5x} \cdot \cos 3x$

Ex:- $y = 5\cos x - 3\sin x$ → prove :- $\frac{d^2y}{dx^2} + y = 0$

Soln:- diff w.r.t → x

$$\frac{dy}{dx} = -5\sin x - 3\cos x$$

→ Diff again wrt x

→ $\frac{d^2y}{dx^2} = -5\cos x + 3\sin x$

⇒ $\frac{d^2y}{dx^2} = -(5\cos x - 3\sin x)$

→ $\frac{d^2y}{dx} = -y \Rightarrow \frac{d^2y}{dx^2} + y = 0$ H.P.

$$\frac{d^2y}{dx^2} + y = 0$$

$$-5\cos x + 3\sin x + 5\cos x - 3\sin x = 0$$

$0 = 0$ H.P.

Ex:- $y = \cos^{-1}x$ → find $\frac{dy}{dx}$ in terms of y.

→ Diff → $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} = \frac{-1}{(1-x^2)^{1/2}} = -(1-x^2)^{-1/2}$

$\Rightarrow -\left(\frac{-1}{2}\right)(1-x^2)^{\frac{-1}{2}-1} \times (0-2x)$

$\Rightarrow \frac{1}{2}(1-x^2)^{-3/2} \times -2x$

$= \frac{-x}{(1-x^2)^{3/2}}$

→ Diff → $\frac{d^2y}{dx^2} = \frac{\sqrt{1-x^2}\,(0) - (-1) \times \frac{1}{2\sqrt{1-x^2}} \times -2x}{(1-x^2)}$

$\left[\frac{d^2y}{dx^2} = \frac{1}{(1-x^2)}\left[\frac{-x}{\sqrt{1-x^2}}\right] = \frac{-x}{(1-x^2)^{3/2}}\right]$

$y = \cos^{-1}x \Rightarrow \cos y = x$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\cos y}{(1-\cos^2 y)^{3/2}} = \frac{-\cos y}{(\sin^2 y)^{3/2}} = \frac{-\cos y}{\sin^3 y} = \frac{-\cos y}{\sin y \cdot \sin^2 y} = -\cot y \cdot \operatorname{cosec}^2 y$

Ex1- H.W $\left[y = (A)e^{mx} + (B)e^{nx} \right] \rightarrow$ Show $\rightarrow \left[\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0 \right]$

Ex:- $y = (\tan^{-1}x)^2$

Show $\rightarrow \left[(x^2+1)^2 y_2 + 2x(x^2+1) y_1 = 2 \right]$

Soln- $\frac{dy}{dx} = y_1 = 2\cdot(\tan^{-1}x) \times \frac{1}{1+x^2}$

$\rightarrow \left[\frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2} \right] \Rightarrow (y_1) = \left[y_1(1+x^2) = 2\tan^{-1}x \right]$

$\rightarrow$ Diff. again $\rightarrow y_1(2x) + (1+x^2)\cdot y_2 = 2\cdot\frac{1}{1+x^2}$

$\Rightarrow \left[y_1 2x(1+x^2) + (1+x^2)^2 \cdot y_2 = 2 \right]$

Ex 1:- $e^y(x+1) = 1$ → show → $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ → diff again

Ex:- $y = 3\cos(\log x) + 4\sin(\log x)$

show → $x^2 \cdot y_2 + xy_1 + y = 0$

Soln: $y_1 = -3\sin(\log x) \times \frac{1}{x} + 4\cos(\log x) \times \frac{1}{x}$

$x \cdot y_2 + y_1(1) =$

→ $x \cdot y_1 = -3\sin(\log x) + 4\cos(\log x)$

→ Diff → $x \cdot y_2 + y_1 = -3\cos(\log x) \times \frac{1}{x} + 4 \cdot (-\sin(\log x)) \times \frac{1}{x}$

$\Rightarrow x y_2 + y_1 = \frac{-3\cos(\log x) - 4\sin(\log x)}{x}$

$\Rightarrow x^2 y_2 + x y_1 = -[3\cos(\log x + 4\sin(\log x)]$

$\rightarrow x^2 y_2 + x y_1 = -y$

Ex:- $y = 5\cos e^{7x} + 6\cos e^{-7x}$

Show $\rightarrow \boxed{\frac{d^2y}{dx^2} = 49y}$

Ex:- Find $\frac{dy}{dx}$ $\rightarrow$ $y = \sin^{-1}(x\sqrt{x}) = ?$

Ex:- $\cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$

Find $\frac{dy}{dx} = ?$