If $y = 2^{-x}$, then $\frac{dy}{dx}$ is equal to :

(a) $-\frac{x}{2^{x+1}}$ $\log M^N$

(b) $2^x \log 2$

(c) $2^{-x} \log 2$ $N \log M$

(d) $\frac{\log \frac{1}{2}}{2^x}$

(-1)

Sol $\rightarrow$ $y = 2^{-x}$

$\frac{dy}{dx} = 2^{-x} \cdot \log 2 \times (-1)$

$\frac{dy}{dx} = 2^{-x} \left[-\log 2 \right]$

$\frac{dy}{dx} = 2^{-x} \left[\log (2)^{-1} \right]$

$\frac{dy}{dx} = \frac{1}{2^x} \left(\log \left(\frac{1}{2} \right) \right)$

If a function f(x) is defined as

$$f(x) = \begin{cases} \dfrac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{then :}$$

(a) f(x) is continuous at x = 0 but not differentiable at x = 0

(b) f(x) is continuous as well as differentiable at x = 0

(c) f(x) is discontinuous at x = 0

(d) None of these.

Sol^n -

$f(x) =$

$\rightarrow RHL = \lim_{x \to 0^+} \dfrac{x}{\sqrt{x^2}} = \dfrac{x}{x} = 1$

$\boxed{LHL = 1}$

$\rightarrow f(0) = 0$

Let $f(x) = \dfrac{\ln(1+ax) - \ln(1-bx)}{x}, x \neq 0$

If f(x) is continuous at $x = 0$, then $f(0) =$

(a) $a - b$ (b) $a + b$

(c) $b - a$ (d) $\ln a + \ln b$

log = ln

$\rightarrow f(0) = \dfrac{\ln(1+0) - \ln(1-0)}{0} = \dfrac{0}{0}$ (undefine)

$\Rightarrow$ L' Hopital Rule:$\rightarrow$ $f(x) = \dfrac{1}{1+ax} \times a - \dfrac{1}{1-bx} \times (-b)$

$f(x) = \dfrac{a}{1+ax} + \dfrac{b}{1-bx}$

$\Rightarrow f(0) = \dfrac{a}{1+0} + \dfrac{b}{1-0} = a+b$

If $f(x) = 2x$ and $g(x) = \frac{x^2}{2} + 1$, then which of the following can be a discontinuous function?

(a) $f(x) + g(x)$

(b) $f(x) - g(x)$

(c) $f(x).g(x)$

(d) $\frac{g(x)}{f(x)}$

$$\frac{\frac{x^2}{2}+1}{2x} = \frac{0}{0}$$

Consider the following statements:

I. The function f(x) = greatest integer ≤ x, x ∈ R is a continuous function.

$f(x) = [x] \to R \to$ Cont

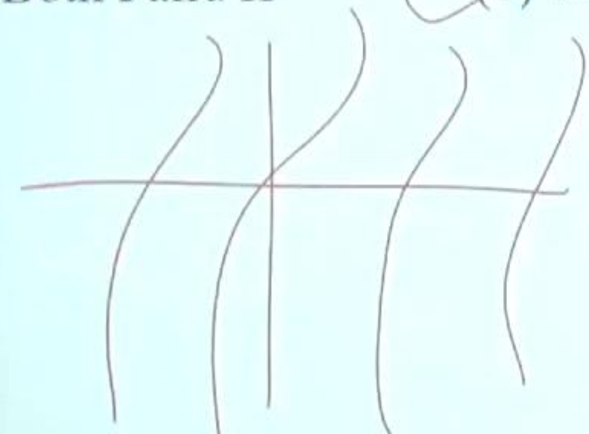
II. All trigonometric functions are continuous on R.

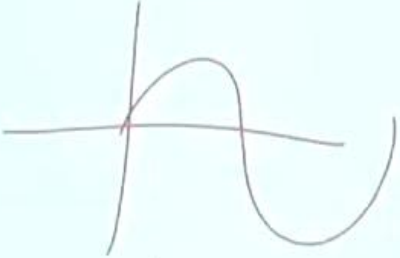
sin θ, cos θ] Cont

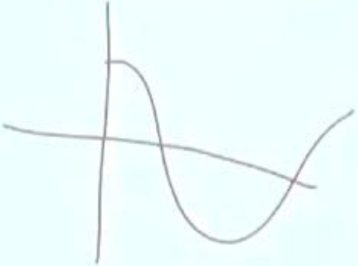
Which of the statements given above is/are correct?

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II







If $f(x) = \begin{cases} \dfrac{\sin 5x}{x^2+2x}, & x \neq 0 \\ k + \dfrac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(a) 1 $\qquad \lim_{h\to 0} \frac{\sin h}{h} = 1$

(b) -2

(c) 2

(d) $\frac{1}{2}$

$\rightarrow$ Limit $= f(0)$

$\frac{5}{2} = k + \frac{1}{2} \rightarrow k = \cancel{\frac{4}{2}}\ 2$

$\rightarrow \lim_{x\to 0^+} f(x) \Rightarrow \lim_{x\to 0^+} \frac{\sin 5x}{x^2+2x}$

$\Rightarrow$ put $x = x + h$, $x = 0$ and $h \to 0$

$\Rightarrow \lim_{h\to 0} \frac{\sin 5(x+h)}{(x+h)^2 + 2(x+h)} \Rightarrow \lim_{h\to 0} \frac{\sin 5h}{h^2+2h} = \lim_{h\to 0} \frac{\sin 5h}{h(h+2)}$

$\Rightarrow \left[\lim_{h\to 0} \frac{\sin 5h}{5h} \times \frac{1 \times 5}{h+2}\right] \Rightarrow \lim_{h\to 0} \frac{5}{h+2} = \frac{5}{2}$

The derivative of e^{x^3} with respect to $\log x$ is

$$\boxed{y = e^{x^3}}$$

diff w.r.t. v $\frac{dy}{dv} \rightarrow dy/dx \,/\, dv/dx$

(a) e^{x^3} (b) $3x^2 2e^{x^3}$

(c) $3x^3 e^{x^3}$ (d) $3x^3 e^{x^3} + 3x^2$

Soln:- $y = e^{x^3}$ $v = \log x$

$\frac{dy}{dx} = 3x^2 \cdot e^{x^3}$ $\frac{dv}{dx} = \frac{1}{x}$

$\Rightarrow \frac{dy}{dv} = \frac{dy/dx}{dv/dx} = \frac{3x^2 \cdot e^{x^3}}{1/x} = 3x^3 \cdot e^{x^3}$

Let $3f(x) - 2f(1/x) = x$, then $f'(2)$ is equal to

(a) $\frac{2}{7}$ (b) $\frac{1}{2}$

(c) 2 (d) $\frac{7}{2}$

$3f(x) - 2f\left(\frac{1}{x}\right) = x$

$\Rightarrow$ Replace $\rightarrow x$ with $1/x$

$\Rightarrow 3f\left(\frac{1}{x}\right) - 2f(x) = \frac{1}{x}$

$\rightarrow f(x) = p \quad \rightarrow f(1/x) = q$

$\Rightarrow 3p - 2q = x \times 3$

$-2p + 3q = \frac{1}{x} \times 2$

$\Rightarrow 9$

$9p - 6q = 3x$

$-4p + 6q = 2/x$

$5p = 3x + \frac{2}{x}$

$p = \frac{3}{5}x + \frac{2}{5x} = f(x)$ → Diff

$\rightarrow f'(x) = \frac{3}{5}(1) + \frac{2}{5}\left(\frac{-1}{x^2}\right)$

$\rightarrow f'(2) = \frac{3}{5} + \frac{\cancel{2}}{5}\left(\frac{-1}{\cancel{4}\,2}\right)$

$= \frac{3}{5} - \frac{1}{10} = \frac{6-1}{10} = \frac{\cancel{5}}{\cancel{10}} \; \frac{1}{2}$

$$f(x) = \begin{cases} x \sin 1/x, & x \neq 0 \\ 0, & x = 0 \end{cases}$$ at $x = 0$ is

(a) continuous as well as differentiable

(b) differentiable but not continuous

(c) continuous but not differentiable

(d) neither continuous nor differentiable

$$\lim_{h \to 0} (x+h) \sin\left(\frac{1}{x+h}\right)$$

$$\lim_{h \to 0} (h) \cdot \sin\left(\frac{1}{h}\right)$$

$$\Rightarrow 0 \times \sin\left(\frac{1}{h}\right) = 0$$

$\underline{\text{Conti}} \rightarrow \text{limit} = f(0)$

$\boxed{0 = 0}$

$$\rightarrow \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} x \cdot \sin\left(\frac{1}{x}\right)$$

$(x = x + h) \rightarrow x = 0 \rightarrow h \rightarrow 0$

$$\frac{\sin h}{h}$$