Conti. & Diff.

Ex:- $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ $\left[\begin{matrix} 2\cos^2 t - 1 \\ 1 - 2\sin^2 t \end{matrix}\right.$

Sol^n:

→ diff x & y wrt → t

$$\Rightarrow \frac{dx}{dt} = \left[\frac{\sqrt{\cos 2t}\cdot 3\sin^2 t \times \cos t - \sin^3 t \cdot \frac{1}{\sqrt{\cos 2t}} \times -\sin 2t \times 2}{(\cos 2t)}\right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{1}{\cos 2t}\left[\frac{\cos 2t \cdot 3\sin^2 t \cdot \cos t + \sin^3 t \cdot \sin 2t}{\sqrt{\cos 2t}}\right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{\cos 2t \cdot 3\sin^2 t \cdot \cos t + \sin^3 t \cdot 2\sin t \cdot \cos t}{(\cos 2t)^{3/2}}\right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{\sin^2 t \cos t (3\cos 2t + 2\sin^2 t)}{(\cos 2t)^{3/2}}\right]$$

$$\rightarrow \frac{dx}{dt} = \left[\frac{\sin^2 t \cdot \cos t \left[3(1 - 2\sin^2 t) + 2\sin^2 t\right]}{(\cos 2t)^{3/2}}\right]$$

$$\left[\frac{dx}{dt} = \frac{\sin^2 t \cdot \cos t \left[3 \cdot 4\sin^2 t\right]}{(\cos 2t)^{3/2}}\right] \quad (1)$$

Conti. & Diff#

$$y = \frac{\cos^3 t}{\sqrt{\cos 2t}} \quad \begin{cases} 2\cos^2 t - 1 \\ 1 - 2\sin^2 t \end{cases}$$

$$\rightarrow \frac{dy}{dt} = \frac{1}{\cos 2t}\left[\sqrt{\cos 2t}\cdot 3\cos^2 t\cdot(-\sin t) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} \times (-\sin 2t) \times 2\right]$$

$$\rightarrow \frac{dy}{dt} = \frac{1}{\cos 2t}\left[\frac{-\cos 2t\cdot 3\cos^2 t\cdot \sin t + \cos^3 t \cdot \sin 2t}{\sqrt{\cos 2t}}\right]$$

$$\rightarrow \frac{dy}{dt} = \frac{1}{(\cos 2t)^{3/2}}\left[-\cos 2t\cdot 3\cos^2 t\cdot \sin t + \cos^3 t\cdot 2\sin t\cos t\right]$$

$$\rightarrow \frac{dy}{dt} = \frac{1}{(\cos 2t)^{3/2}}\left[\sin t\cdot\cos^2 t\left[-\cos 2t\cdot 3 + 2\cos^2 t\right]\right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{(\cos 2t)^{3/2}}\left[\sin t\cdot\cos^2 t\cdot\left[-3(2\cos^2 t - 1) + 2\cos^2 t\right]\right]$$

$$\rightarrow \boxed{\frac{dy}{dt} = \frac{1}{(\cos 2t)^{3/2}}\left[\sin t\cos^2 t\cdot(-4\cos^2 t + 3)\right]} \quad -(2)$$

$\rightarrow$ Now $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} =$

$$\frac{dy}{dx} = \frac{\sin t\cdot\cos^2 t\cdot(3 - 4\cos^2 t)}{\sin^2 t\cdot\cos t\,(3 - 4\sin^2 t)}$$

$$\frac{dy}{dx} = \frac{\cos t\,(3 - 4\cos^2 t)}{\sin t\,(3 - 4\sin^2 t)} = \frac{3\cos t - 4\cos^3 t}{3\sin t - 4\sin^3 t}$$

$$\frac{dy}{dx} = \frac{-(4\cos^3 t - 3\cos t)}{\sin 3t}$$

$$\frac{dy}{dx} = \frac{-\cos 3t}{\sin 3t} = -\cot 3t$$ Ans

$$\boxed{\frac{dx}{dt} = \frac{\sin^2 t\cdot\cos t\,(\quad - 4\sin^2 t]}{(\cos 2t)^{3/2}}} \quad -(1)$$

$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Conti. & Diff.

Ex:- $x = a(\cos t + \log|\tan t/2|)$, $y = a\sin t$

$\left[\sin 2\theta = 2\sin\theta\cos\theta \quad \sin\theta = 2\sin\theta/2 \cdot \cos\theta/2\right]$

→ diff. wrt → t

$$\Rightarrow \frac{dx}{dt} = a\left[-\sin t + \frac{1}{\tan t/2} \times \sec^2 t/2 \times \frac{1}{2}\right] = a\left[-\sin t + \frac{1}{2}\cdot\frac{\cos t/2}{\sin t/2} \times \frac{1}{\cos^2 t/2}\right]$$

$$\Rightarrow \frac{dx}{dt} = a\left[-\sin t + \frac{1}{2\cdot\sin t/2\cdot\cos t/2}\right] = a\left[-\sin t + \frac{1}{\sin t}\right] = a\left[\frac{-\sin^2 t + 1}{\sin t}\right]$$

$$\rightarrow \frac{dx}{dt} = a\left[\frac{\cos^2 t}{\sin t}\right] \quad -(I)$$

$$\rightarrow \frac{dy}{dt} = a(\cos t) \quad -(II)$$

$$\rightarrow \frac{dy}{dx} = \frac{a\cos t}{a\cdot\frac{\cos^2 t}{\sin t}} = \frac{\sin t}{\cos t} = \tan t.$$

Conti. & Dif.

Ex:- Hw. $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$ → $\frac{dy}{dx} = -\frac{y}{x}$

$x = \left(a^{\sin^{-1}t}\right)^{1/2}$ $\{ (a^m)^n = a^{m \cdot n} \}$

$x = a^{\frac{1}{2}\cdot \sin^{-1}t}$

→ log → $\log x = \log a^{\frac{1}{2}\cdot \sin^{-1}t}$

$\log x = \frac{1}{2} \sin^{-1}t \cdot \log a$

Ex:- $x = a(\cos\theta + \theta \sin\theta)$
$y = a(\sin\theta - \theta \cdot \cos\theta)$ > dy/dx

Ex:- $x = a\sec\theta$
$y = b\tan\theta$ > dy/dx

Ex: $x = \cos\theta - \cos 2\theta$
$y = \sin\theta - \sin 2\theta$

Sol:- $\frac{dx}{d\theta} = (-\sin\theta + \sin 2\theta \times 2)$

$\frac{dy}{d\theta} = \cos\theta - \cos 2\theta \times 2 = \cos\theta - 2\cos 2\theta$

$\frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$ Ans

$x^y + y^x = 1$

$= \frac{\cos\theta - 2\cos 2\theta}{\sin\theta(4\cos\theta - 1)}$ Ans

Conti. & Diff.

Ex: If $x^y + y^x = 1$

Let $u = x^y$

$\log u = y \cdot \log x$

$\frac{1}{u} \cdot \frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$

$\frac{du}{dx} = u\left[\frac{y}{x} + \log x \cdot \frac{dy}{dx}\right] - (1)$

$v = y^x \Rightarrow \log v = x \cdot \log y$

$\frac{1}{v} \cdot \frac{dv}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1$

$\frac{dv}{dx} = v\left[\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right]$

$\Rightarrow u + v = 1 \rightarrow$ diff $\rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$

$\Rightarrow$

$\cos^{-1}t \rightarrow \frac{dy}{dx} = -\frac{y}{x}$

$a^m)^n = a^{m \cdot n}$

Ex: $x = \cos\theta - \cos 2\theta$

$y = \sin\theta - \sin 2\theta$

Sol:-

$\frac{dx}{d\theta} = -\sin\theta + \sin 2\theta \times 2$

$\frac{dy}{d\theta} = \cos\theta - \cos 2\theta \times 2 = \cos\theta - 2\cos 2\theta$

$\frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$ Ans

$= \frac{\cos\theta - 2\cos 2\theta}{\sin\theta(4\cos\theta - 1)}$ Ans