Conti. & Diff.

Ex:- (1) $x^y = e^{(x-y)}$

→ take log :-

$\Rightarrow \log x^y = \log e^{(x-y)}$

$\Rightarrow y \log x = (x-y) \cdot \log_e e$

$\Rightarrow y \log x = (x-y)$

→ diff wrt x.

$\Rightarrow \frac{y}{x} = 1 - \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = 1$

$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y} + 1\right] = \frac{x-1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{(x-1)}{x} \times \frac{y}{1+y} = \frac{y(x-1)}{x(1+y)}$ Ans

H.W Ex:- $x^y + y^x = 1$ → Sol → $\frac{y \cdot x^{y-1} + y^x \log y}{x^y \log x + x \cdot y^{x-1}}$ (H.W)

Ex:- $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$

→ find $f'(x)$ & $f'(1) = ?$

↓

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Conti. & Diff.

Ex:- $(x^2-5x+8)(x^3+7x+9) = y$ $\Rightarrow \log y = \log(x^2-5x+8) + \log(x^3+7x+9)$

$\rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x^2-5x+8}\times 2x-5+\frac{1}{x^3+7x+9}\times 3x^2+7$

i) Product Rule

ii) expending product to obtain a single polynomial.

iii) logarithmic diff.

$\frac{dy}{dx} = y\left[\frac{2x-5}{x^2-5x+8} + \frac{3x^2+7}{x^3+7x+9}\right]$

Sol:- i) $y = (x^2-5x+8)(x^3+7x+9)$

$\frac{dy}{dx} = y\left[\frac{(2x-5)(x^3+7x+9)+(3x^2+7)(x^2-5x+8)}{(x^2-5x+8)(x^3+7x+9)}\right]$

$\Rightarrow \frac{dy}{dx} = (x^2-5x+8)(3x^2+7+0)+(x^3+7x+9)(2x-5+0)$

$\rightarrow \frac{dy}{dx} = 3x^4+7x^2-15x^3-35x+24x^2+56+2x^4-5x^3+14x^2-35x+18x+45$

$\rightarrow \boxed{\frac{dy}{dx} = 5x^4+45x^2-20x^3-52x+11}$ ✓

ii) $y = x^5+7x^3+9x^2-5x^4-35x^2-45x+8x^3+56x+72$

$y = x^5+15x^3-5x^4-26x^2+11x+72$

$\rightarrow \boxed{\frac{dy}{dx} = 5x^4+45x^2-20x^3-52x+11}$ ✓

iii) $\log y = \log[(x^2-5x+8)\cdot(x^3+7x+9)]$

$\frac{dy}{dx} = y\cdot\frac{2x^4+14x^2+18x-5x^3-35x-45+3x^4-15x^3+24x^2+7x^2-35x+56}{y}$

$\boxed{\frac{dy}{dx} = 5x^4-20x^3+45x^2-52x+11}$ ✓

Conti. & Diff.

Ex 1:- $x = a\cos\theta$, $y = b\cos\theta$ $\rightarrow \frac{dy}{dx}$

$\Rightarrow$ diff. wrt θ :-

$\rightarrow \frac{dx}{d\theta} = a(-\sin\theta)$, $\frac{dy}{d\theta} = b(-\sin\theta)$

$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{b(-\sin\theta)}{a(-\sin\theta)} = b/a$

Ex:- $x = 4t$, $y = 4/t$ $\rightarrow \frac{dy}{dx} = -\frac{1}{t^2}$

Ex:- $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$

$\rightarrow$ diff wrt θ

$\frac{dx}{d\theta} = a[1-\cos\theta]$, $\frac{dy}{d\theta} = a[0+(-\sin\theta)]$

$\rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-a\sin\theta}{a(1-\cos\theta)} = -\frac{\sin\theta}{(1-\cos\theta)}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow 2\sin^2\theta = 1-\cos 2\theta$

$\Rightarrow \frac{dy}{dx} = -\frac{2\sin\theta/2\cos\theta/2}{2\cdot\sin^2\theta/2} = -\frac{\cos\theta/2}{\sin\theta/2}$

$\rightarrow \frac{dy}{dx} = -\cot\theta/2$ ✓

H.W.

Ex:- $x = \cos\theta - \cos 2\theta$

$y = \sin\theta - \sin 2\theta$

Ex:- $x = a\left(\cot t + \log\tan t/2\right)$, $y = a\sin t$

Soln $\rightarrow \tan t$