Conti. & Diff.

Ex :- $y = \sqrt{e^{\sqrt{x}}}$

Solⁿ :-

$$\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}} \times \frac{d}{dx}\left(e^{\sqrt{x}}\right)$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} \times e^{\sqrt{x}} \times \frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} \times e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{4\sqrt{x \cdot e^{\sqrt{x}}}} \times e^{\sqrt{x}} \quad Ans.$$

H.W. i) $\frac{\cos x}{\log x}$ → diff. → w.r.to x

ii) $\cos(\log x + e^x)$

Conti. & Diff.

Ex:- diff. wrt $x \rightarrow \cos x \cdot \cos 2x \cdot \cos 3x$

Soln:. let $y = \cos x \cdot \cos 2x \cdot \cos 3x$

$\rightarrow$ take log on both:-

$\Rightarrow \log y = \log(\cos x \cdot \cos 2x \cdot \cos 3x)$

$\Rightarrow \log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)$

$\Rightarrow$ Now Diff.

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{\cos x} \times (-\sin x) + \frac{1 \times 2 \times (-\sin 2x)}{\cos 2x} + \frac{1}{\cos 3x} \times (-\sin 3x) \times 3$$

$$\Rightarrow \frac{dy}{dx} = y\left[-\tan x - 2\tan 2x - 3\tan 3x\right]$$

$$\Rightarrow \frac{dy}{dx} = -\cos x \cdot \cos 2x \cdot \cos 3x\left[\tan x + 2\tan 2x + 3\tan 3x\right] \text{ Ans}$$

Conti. & Diff.

Ex:- diff. w.r.t $x \rightarrow \sqrt{\dfrac{(x+1)(x+2)}{(x+3)(x+4)(x+5)}} = y$ (let)

take log on both side:

$\Rightarrow \log y = \log\left[\dfrac{(x+1)(x+2)}{(x+3)(x+4)(x+5)}\right]^{1/2} \rightarrow \log M^N$

$\Rightarrow \log y = \dfrac{1}{2}\log\left[\dfrac{(x+1)(x+2)}{(x+3)(x+4)(x+5)}\right] \rightarrow \log\left(\dfrac{M}{N}\right)$

$\Rightarrow \log y = \dfrac{1}{2}\left\{\log\left[(x+1)(x+2)\right] - \log\left[(x+3)(x+4)(x+5)\right]\right\}$

$\Rightarrow \log y = \dfrac{1}{2}\left\{\log(x+1) + \log(x+2) - \log(x+3) - \log(x+4) - \log(x+5)\right\}$

$\Rightarrow$ diff. w.r.t $\rightarrow x$

$\Rightarrow \dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{1}{2}\left\{\dfrac{1}{x+1}\times 1 + \dfrac{1}{x+2}\times 1 - \dfrac{1}{x+3} - \dfrac{1}{x+4} - \dfrac{1}{x+5}\right\}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{y}{2}\left[\dfrac{1}{x+1} + \dfrac{1}{x+2} - \dfrac{1}{x+3} - \dfrac{1}{x+4} - \dfrac{1}{x+5}\right]$

$\dfrac{dy}{dx} = \dfrac{1}{2}\sqrt{\dfrac{(x+1)(x+2)}{(x+3)(x+4)(x+5)}}\left[\dfrac{1}{x+1} + \dfrac{1}{x+2} - \dfrac{1}{x+3} - \dfrac{1}{x+4} - \dfrac{1}{x+5}\right]$ Ans

H.W

$y = (x+3)^2(x+4)^3(x+5)^4$

Sol $\rightarrow (x+3)(x+4)^2(x+5)^3(9x^2+70x+133)$

Conti. & Diff.

Ex:- diff. wrt $x \rightarrow x^x - 2^{\sin x}$

Soln. let $y = x^x - 2^{\sin x}$ $\quad x^n$

Now let $u = x^x$

$[v = 2^{\sin x}]$

So :- $y = u - v \Rightarrow$ diff. $\rightarrow \left[\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}\right]$ — (i)

$\Rightarrow [u = x^x] \rightarrow \left[\log u = \log(x^x) \Rightarrow \frac{x}{u} \cdot \frac{\log(x)}{v}\right]$

$\rightarrow$ diff wrt $x \rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{x} + \log x.(1)$

$\Rightarrow \frac{du}{dx} = u\left[1 + \log x\right] = x^x\left[1 + \log x\right]$ (ii)

$\rightarrow$ Now $v = 2^{\sin x} \rightarrow$ take log on both side

$\Rightarrow \left[\log v = \log 2^{\sin x} = \frac{\sin x \cdot \log 2}{u \cdot \quad v}\right]$

$\Rightarrow$ diff wrt $\rightarrow x$

$\frac{1}{v} \cdot \frac{dv}{dx} = \left[\sin x \cdot (0) + \log 2(\cos x)\right]$

$\frac{dv}{dx} = v\left[\log 2 \cdot \cos x\right]$

$\frac{dv}{dx} = 2^{\sin x}\left(\log 2 \cdot \cos x\right)$ (3)

from eqn (i) (ii) & (iii)

$\frac{dy}{dx} = x^x\left(1 + \log x\right) - 2^{\sin x} \cdot \log 2 \cdot \cos x$ Ans

$v \Rightarrow 2^{\sin x} \Rightarrow \frac{dv}{dx} = 2^{\sin x} \times \log_e 2 \times \cos x$

Conti. & Diff.

H.W. $\left[y = (\sin x)^x + \sin^{-1}\sqrt{x}\right] \rightarrow$ sol

$$(\sin x)^x\left[x\cot x + \log\sin x\right] + \frac{1}{2\sqrt{x-x^2}}$$

Ex:- diff. w.r.t $x \rightarrow x^{x\cos x} + \frac{x^2+1}{x^2-1} = y$ (let)

let:- $u = x^{x\cos x}$, $v = \frac{x^2+1}{x^2-1}$

so: $y = u + v$

$\rightarrow$ diff-1 $\left[\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}\right]$ — (1)

Now $\boxed{u = x^{x\cos x}}$

take log:- $\log u = \log x^{x\cos x}$

$\Rightarrow \left[\log u = \underset{(i)}{x}\cdot\underset{(ii)}{\cos x}\cdot\underset{(iii)}{\log x}\right]$

diff. $\rightarrow \frac{1}{u}\frac{du}{dx} = (1).\cos x.\log x + x.\log x(-\sin x) + \cancel{x}.\cos x.\frac{1}{\cancel{x}}$

$\Rightarrow \frac{du}{dx} = u\left[\cos x.\log x - x.\sin x.\log x + \cos x\right]$

$\rightarrow \frac{du}{dx} = x^{x\cos x}\left[\cos x.\log x - x\sin x.\log x + \cos x\right]$ — (2)

Now $v = \frac{x^2+1}{x^2-1}$

diff.

$$\frac{dv}{dx} = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$\left[\frac{dv}{dx} = \frac{2\cancel{x^3} - 2x - 2\cancel{x^3} - 2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}\right] \text{ — (3)}$$

$\rightarrow$ take log:- $\log v = \log\left(\frac{x^2+1}{x^2-1}\right)$

$\rightarrow \log v = \log(x^2+1) - \log(x^2-1)$

$\rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = \left[\frac{1}{x^2+1}\times 2x - \frac{1}{x^2-1}\times 2x\right] \Rightarrow \frac{dv}{dx} = v\left[\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right]$

$\Rightarrow \frac{dv}{dx} = \frac{x^2+1}{x^2-1}\times 2x\left[\frac{1}{x^2+1} - \frac{1}{x^2-1}\right]$

$\frac{dv}{dx} = \frac{(\cancel{x^2+1})}{(x^2-1)}\times 2x\left[\frac{\cancel{x^2}-1-\cancel{x^2}-1}{(\cancel{x^2+1})(x^2-1)}\right]$

$\frac{dv}{dx} = -4x/(x^2-1)^2$