Conti & Diff.

Ex:- $2x + 3y = \sin y$

$\Rightarrow 2 + 3\cdot\frac{dy}{dx} = \cos y \times \frac{dy}{dx}$

$\Rightarrow 2 = \cos y \cdot \frac{dy}{dx} - 3\cdot\frac{dy}{dx}$

$\Rightarrow 2 = \frac{dy}{dx}(\cos y - 3)$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{2}{\cos y - 3}}$ Ans

Ex:- $y = \log x + e^x \sin x + \log_5 x$

$\frac{dy}{dx} = \frac{1}{x} + e^x \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(e^x) + \frac{1}{x} \times \frac{1}{\log 5}$

$\boxed{\frac{dy}{dx} = \frac{1}{x} + e^x \cos x + \sin x \cdot e^x + \frac{1}{x}\cdot\frac{1}{\log 5}}$ Ans

Ex:- $y = \log\left(x + \sqrt{a^2 + x^2}\right)$ (x)

$\frac{dy}{dx} = \frac{1}{x + \sqrt{a^2+x^2}} \times \frac{d}{dx}\left(x + \sqrt{a^2+x^2}\right)$

$\frac{dy}{dx} = \frac{1}{x + \sqrt{a^2+x^2}}\left\{\frac{d}{dx}(x) + \left(\frac{d}{dx}\sqrt{a^2+x^2}\right)\right\}$ $\Rightarrow \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{x + \sqrt{a^2+x^2}}\left\{1 + \frac{1}{2\sqrt{a^2+x^2}} \times \frac{d}{dx}(a^2 + x^2)\right.$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x + \sqrt{a^2+x^2}}\left(1 + \frac{2x}{2\sqrt{a^2+x^2}}\right)$

$\frac{dy}{dx} = \frac{1}{x + \sqrt{a^2+x^2}}\left(\frac{\sqrt{a^2+x^2} + x}{\sqrt{a^2+x^2}}\right)$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{a^2+x^2}}}$ Ans

Conti & Diff.

Ex:- if $y = x^2$ → find $\frac{dy}{dx}$ at $x = 2$

$$\left.\frac{dy}{dx}\right|_{x=2} = 2(x) = 2(2) = 4$$

Ex:- if $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ → find $\frac{dy}{dx} = ?$

⇒ put → $x = \tan\theta \Rightarrow [\theta = \tan^{-1}x]$

$$\Rightarrow y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$\Rightarrow y = \sin^{-1}(\sin 2\theta)$$

$$\Rightarrow y = 2\theta = 2\cdot\tan^{-1}x$$

$$\Rightarrow \boxed{y = 2\tan^{-1}x}$$

diff wrt x → $\frac{dy}{dx} = 2\cdot\frac{1}{1+x^2}$ Ans

Ex:- $y = \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right)$

→ put $x = \tan\theta \Rightarrow \theta = \tan^{-1}x$

$$\Rightarrow y = \tan^{-1}\left[\frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}\right]$$

$$y = \tan^{-1}[\tan 3\theta] \Rightarrow y = 3\theta$$

$$y = 3\cdot\tan^{-1}x \Rightarrow \frac{dy}{dx} = 3\cdot\frac{1}{1+x^2}$$ Ans

H.W. Ex:- $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Conti & Diff.

Ex: $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta \rightarrow \theta = \tan^{-1}x$

$\Rightarrow y = \cos^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$

$\Rightarrow y = \cos^{-1}[\sin 2\theta]$

$\Rightarrow y = \cos^{-1}[\cos(90-2\theta)]$

$\Rightarrow y = 90 - 2\theta \Rightarrow 90 - 2\cdot\tan^{-1}x$

$\Rightarrow \frac{dy}{dx} = 0 - 2\cdot\frac{1}{1+x^2}$

$\Rightarrow \boxed{\frac{dy}{dx} = \frac{-2}{1+x^2}}$ ✓

Ex: $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

Ex: $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x$

$\Rightarrow y = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right)$

$\rightarrow y = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow y = \sec^{-1}(\sec 2\theta)$

$\Rightarrow y = 2\theta = 2\cos^{-1}x$

$\Rightarrow \frac{dy}{dx} = 2\cdot\frac{-1}{\sqrt{1-x^2}}$ ✓