Conti. & Diff.

Ex:- $x^3 + x^2y + xy^2 + y^3 = 81$ → find $\frac{dy}{dx} = ?$

Sol:- diff. wrt to x

$$\Rightarrow \frac{d}{dx}(x^3) + \frac{d}{dx}(x^2 y) + \frac{d}{dx}(x y^2) + \frac{d}{dx}(y^3) = \frac{d}{dx}(81)$$

$$\Rightarrow 3x^2 + \left[x^2 \cdot \frac{dy}{dx} + y \cdot (2x)\right] + \left[x \cdot 2y \times \frac{dy}{dx} + y^2 \cdot (1)\right] + 3y^2 \times \frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + 2xy + 2xy \cdot \frac{dy}{dx} + y^2 + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}\left[x^2 + 2xy + 3y^2\right] = -(3x^2 + 2xy + y^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(3x^2 + 2xy + y^2)}{(x^2 + 2xy + 3y^2)}$$

Ex:- $2x + 3y = \sin y$ → $\frac{dy}{dx}$

Ex:- $ax + by^2 = \cos y$ → $\frac{dy}{dx}$

diff wrt x

$$\Rightarrow a + b \cdot 2y \cdot x\frac{dy}{dx} = -\sin y \times \frac{dy}{dx}$$

$$\Rightarrow 2by \cdot \frac{dy}{dx} + \sin y \frac{dy}{dx} = -a$$

$$\Rightarrow \frac{dy}{dx}(2by + \sin y) = -a$$

$$\boxed{\frac{dy}{dx} = \frac{-a}{2by + \sin y}}$$

#Conti. & Diff.

i) $\sin(2x+5)$

Soln:- $y = \sin(2x+5)$

→ diff. w.r.t → x → $\cos(2x+5) \times \frac{d}{dx}(2x+5)$

$\Rightarrow \frac{dy}{dx} = \cos(2x+5) \times (2(1)+0)$

$\rightarrow \frac{dy}{dx} = 2\cos(2x+5)$ ✓

ii) $y = \cos(\sqrt{x})$

→ diff wrt → x

$\Rightarrow \frac{dy}{dx} = -\sin(\sqrt{x}) \times \frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{dy}{dx} = -\sin(\sqrt{x}) \times \left(\frac{1}{2\sqrt{x}}\right)$

$= \frac{dy}{dx} = \frac{-\sin(\sqrt{x})}{2(\sqrt{x})}$ ✓

$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$

3) → $y = (2x+1)^3$

→ $\frac{dy}{dx} = 3.(2x+1)^2 \times \frac{d}{dx}(2x+1)$

$= 3(2x+1)^2 \times (2(1)+0)$

$= 6(2x+1)^2$ ✓

4) $y = \sin(\cos x^2)$ $\quad \frac{d}{dx}(x) = 1$

diff → wrt → x.

$\frac{dy}{dx} = \cos(\cos x^2) \times \frac{d}{dx}(\cos x^2)$

$\frac{dy}{dx} = \cos(\cos x^2) \times -\sin x^2 \times \frac{d}{dx}(x^2)$

$\frac{dy}{dx} = \cos(\cos x^2) \times -\sin x^2 \times 2x$ ✓

5) $y = \frac{\sin(9x+6)}{\cos(cx+d)} \Rightarrow y = \frac{u \to x}{v \to x}$

$\frac{dy}{dx} = \frac{\cos(cx+d) \cdot \cos(9x+6) \times 9(1) - \sin(9x+6) \cdot [-\sin(cx+d)] \cdot c(1)}{[\cos(cx+d)]^2}$

$\frac{dy}{dx} =$

Conti. & Diff.

Ex 1. $\sin^2 y + \cos xy = \pi$ $\rightarrow \frac{d}{dx}(a^x) = a^x \log_e a$

Sol:- diff wrt $\rightarrow x$

$\Rightarrow \frac{d}{dx}(\sin^2 y) + \frac{d}{dx}(\cos xy) = \frac{d}{dx}(\pi)$

$\Rightarrow 2\sin y \times \frac{d}{dx}(\sin y) + \left[-\sin(xy) \times \frac{d}{dx}(xy)\right] = 0$

$\Rightarrow 2\sin y \times \cos y \times \frac{d}{dx}(y) + \left[-\sin xy \times \left\{x \cdot \frac{dy}{dx} + y(1)\right\} = 0\right.$

$\Rightarrow 2\sin y \cos y \frac{dy}{dx} - x \cdot \sin xy \cdot \frac{dy}{dx} - y \sin xy = 0$

$\Rightarrow \frac{dy}{dx}\left[2\sin y \cos y - x \cdot \sin xy\right] = y \sin xy$

$\Rightarrow \frac{dy}{dx} = \frac{y \sin xy}{2\sin y \cos y - x \sin xy} = \frac{y \sin xy}{\sin 2y - x \sin xy}$ Ans

H.W

Ex 1:- $y = \log\left(x + \sqrt{a^2 + x^2}\right) \rightarrow a$ Constant

H.W

Ex 1:- $y = \log x + e^x \cdot \sin x + \log_5 x$

Ex 2:- $y = \frac{x \cdot \tan x}{2^x}$

$\frac{dy}{dx} = \frac{2^x \cdot \frac{d}{dx}(x \cdot \tan x) - (x \tan x)\frac{d}{dx}(2^x)}{(2^x)^2}$

$= \frac{2^x\left[x \cdot \sec^2 x + \tan x\right] - x \tan x \, 2^x \log 2}{(2^x)^2}$

$\Rightarrow \frac{x \cdot 2^x \cdot \sec^2 x + 2^x \cdot \tan x - x \cdot \tan x \cdot 2^x \cdot \log 2}{(2^x)^2}$ Ans