

Let $X = \{-1, 0, 1\}$, $Y = \{0, 2\}$ and a function $f: X \rightarrow Y$ defined by $y = 2x^2$, is

- (a) one-one onto
- (b) one-one into
- (c) many-one onto
- (d) many-one into

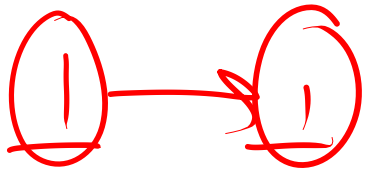
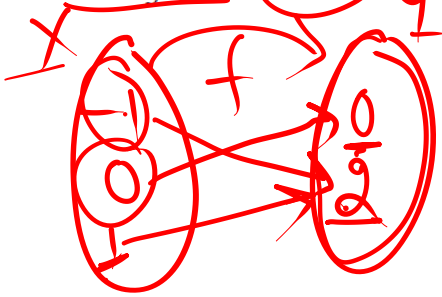


$$y = 2x^2$$

$$y = 2(-1)^2 = 2$$

$$y = 2(1)^2 = 2$$

$$y = 2(0)^2 = 0$$

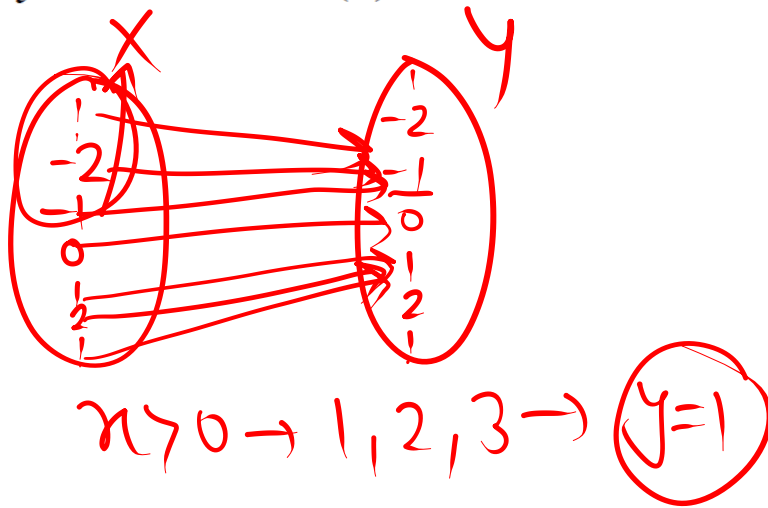


one-one

The signum function, $f: \mathbb{R} \rightarrow \mathbb{R}$ is given by

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

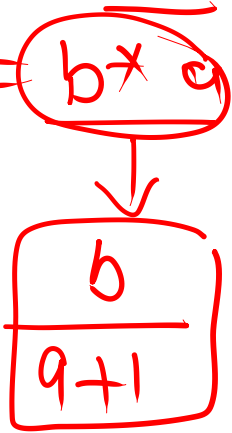
- (a) one-one
- (b) onto
- (c) many-one
- (d) None of these



For binary operation $*$ defined on $\mathbb{R} -$

$\{1\}$ such that $a * b = \frac{a}{b+1}$ is $\neq b * a$

- (a) not associative
- (b) not commutative
- (c) commutative
- (d) both (a) and (b)



$$\begin{aligned}
 & \underline{(a * b) * c} = a * \underline{(b * c)} \\
 & \left(\frac{a}{b+1} \right) * c = a * \left(\frac{b}{c+1} \right) \\
 & \frac{\frac{a}{b+1}}{c+1} = \frac{a}{(b+1)(c+1)} \neq \frac{a}{\frac{b}{c+1} + 1}
 \end{aligned}$$

The diagram shows the calculation of $(a * b) * c$ and $a * (b * c)$. The first calculation results in $\frac{a}{(b+1)(c+1)}$. The second calculation results in $\frac{a}{\frac{b}{c+1} + 1}$. A red arrow points from the first result to the second, indicating that they are not equal, thus proving the operation is not associative.

Given $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) =$

$\frac{3x+x^3}{1+3x^2}$, then $f \circ g(x)$ equals

- (a) $-f(x)$
- (b) $3f(x)$
- (c) $[f(x)]^3$
- (d) None of these

$\log M^N = N \log M$

$f[g(x)] \Rightarrow f\left[\frac{3x+x^3}{1+3x^2}\right] = f(x)$

$f \circ g(x) \Rightarrow f\left[\frac{3x+x^3}{1+3x^2}\right] = \log\left[\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right]$

$= \log\left[\frac{1+(3x^2)+(3x+x^3)}{1+3x^2-3x-x^3}\right]$

$= \log\left[\frac{(1+x)^3}{(1-x)^3}\right]$

$= \log\left[\frac{1+x}{1-x}\right]^3 = 3 \log\left[\frac{1+x}{1-x}\right] = 3f(x)$

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
 $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
 $(\frac{1+x}{1-x})^3 = \frac{a^3 + b^3 + 3a^2b + 3ab^2}{a^3 - b^3 - 3a^2b + 3ab^2}$

Let $f: \mathbb{R} - \left\{ \frac{3}{5} \right\} \rightarrow \mathbb{R}$ be defined by $f(x)$

$= \frac{3x+2}{5x-3}$. Then

$f(x) = \frac{3x+2}{5x-3} = y$

(a) $f^{-1}(x) = f(x)$

$\Rightarrow 3x+2 = 5xy-3y$

(b) $f^{-1}(x) = -f(x)$

$\Rightarrow 3y+2 = 5xy-3x$

(c) $(f \circ f)x = -x$

(d) $f^{-1}(x) = \frac{1}{19} f(x)$

$= 3y+2 = x(5y-3)$

$x = \frac{3y+2}{5y-3}$

$f^{-1}(y)$

$f^{-1} \rightarrow$ inverse of f

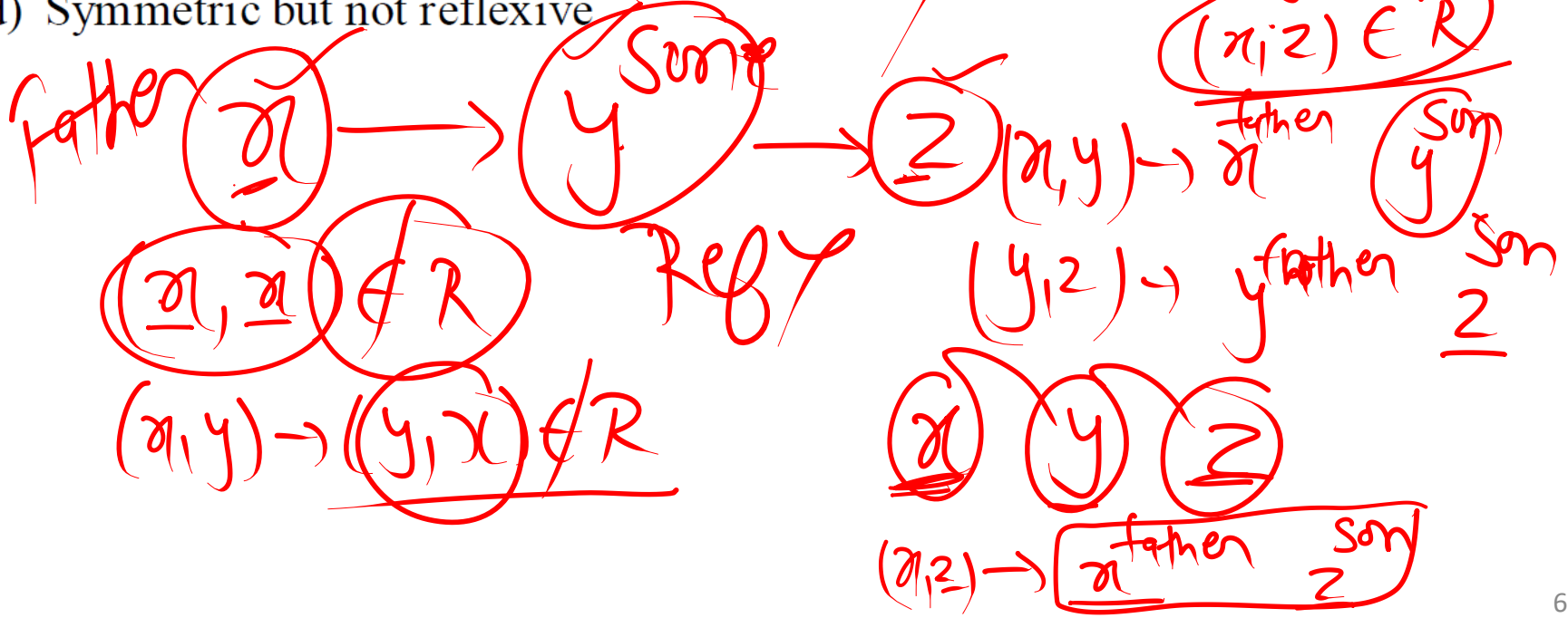
$f(x) \rightarrow f^{-1}(a)$

If $R = \{(x, y) : x \text{ is father of } y\}$, then R is

- (a) reflexive but not symmetric
- (b) symmetric and transitive
- (c) neither reflexive nor symmetric nor transitive
- (d) Symmetric but not reflexive

$(x, y) \in R$
 $(y, z) \in R$
 \downarrow
 $(x, z) \notin R$

And
 $(x, y) \in R \mid (y, z) \in R$
 \downarrow
 $(x, z) \in R$



Which of the following functions from \mathbb{Z} into \mathbb{Z} are bijective?

(a) $f(x) = x^3$

(b) $f(x) = x + 2$

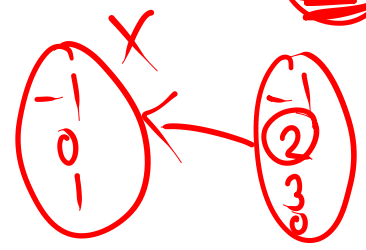
(c) $f(x) = 2x + 1$

(d) $f(x) = x^2 + 1$

one-one + onto

$x = \frac{y-1}{2} = \frac{1}{2}$

$y = x^3 \Rightarrow x = \sqrt[3]{y} = \sqrt[3]{8}$



Which of the following is not a binary operation on the indicated set?

- (a) On \mathbb{Z}^+ , * defined by $a * b = a - b$
- (b) On \mathbb{Z}^+ , * defined by $a * b = ab$
- (c) On \mathbb{R} , * defined by $a * b = ab^2 = -2(4) = (-8)$
- (d) None of the above

$a = 2, b = 1 \rightarrow a * b = 2 - 1 = 1 \in \mathbb{Z}^+$

$a = 1, b = 2 \Rightarrow 1 - 2 = -1 \notin \mathbb{Z}^+$

$2, 2 \Rightarrow a * b = 4 \in \mathbb{Z}^+$ $\frac{a=2}{b=-2}$

$3, 6 = 18 \in \mathbb{Z}^+$

$a = 2, b = -1 \Rightarrow 2 \in \mathbb{R}$

Consider the following statements

I. Addition, subtraction and multiplication are binary operations on \mathbb{R} .

II. Division is a binary operation on \mathbb{R} but not a binary operation on non-zero real numbers.

- (a) Only I is true
- (b) Only II is true
- (c) Both I and II are true
- (d) Neither I nor II is true

$a + b = a + b$ ✓
 $a - b = a - b$ ✓
 $a \cdot b = a \cdot b$ ✓

$a * b = a / b$
 $5 / 0 = \infty$
 $\mathbb{R} \Rightarrow a = 5, b = 0$
 $5 / 0 \notin \mathbb{R}$

1	2	3	4	5	6	7	8	9	10
C	C	D	B	A	C	B	A	A	C