ABLES® KOTA

Conti. & diff.

Ex:- $f(x) = x^2 - \sin x + 5$ is Conti at $x=\pi$ or not?

Sol^n:- $\because f(x) \to$ conti $\to$ if $\lim_{x\to\pi} f(x) = f(\pi)$

$\Rightarrow f(\pi) = \pi^2 - \sin\pi + 5 = \pi^2 + 5 = f(\pi)$

$\because \lim_{x\to\pi} f(x) = \lim_{x\to\pi}[x^2 - \sin x + 5]$

let put $x = \pi + h \Rightarrow x\to\pi$ & $h\to 0$

$\Rightarrow \lim_{x\to\pi}[(\pi+h)^2 - \sin(\pi+h) + 5]$

$\Rightarrow \lim_{h\to 0}[(\pi+h)^2 - \sin(\pi+h) + 5]$

$\Rightarrow (\pi+0)^2 - \sin(\pi+0) + 5$

$\Rightarrow \pi^2 - \sin\pi + 5 = \pi^2 + 5$

$\because$ Limit $= f(\pi)$ so $f(x) \to$ Conti.

Ex:- find a & b such that

$f(x) = \begin{cases} 5 & ; x \le 2 \\ ax+b & ; 2 < x < 10 \\ 21 & ; x \ge 10 \end{cases}$ is Conti.

Ex:- $f(x) = \underbrace{|x|}_{g(x)} - \underbrace{|x+1|}_{h(x)}$ $0 = x = -1$

i) $g(x) = \begin{cases} x & ; x \ge 0 \\ -x & ; x < 0 \end{cases} \to$ LHL = RHL = f(0)

$\Rightarrow \lim_{x\to 0^-}(-x) = \lim_{x\to 0^+}(x) = f(0)$

$\Rightarrow \boxed{0 = 0 = 0} \to g(x)$ Conti

$\boxed{x < -1}$

$x > -1$

$\Rightarrow h(x) = \begin{cases} x+1 & ; x \ge -1 \\ -(x+1) & ; x < -1 \end{cases}$

Conti. & diff.

Differentiability :- a function $f(x)$ is diffentiable at a point c in a real no.

if $\boxed{RHD = LHD}$ ⇒ finite and equal

$$\Rightarrow \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f(x-h)-f(x)}{-h}$$

$$\Rightarrow \lim_{h\to 0} \frac{f(c+h)-f(c)}{h} = \lim_{h\to 0} \frac{f(c-h)-f(c)}{-h}$$

Ques: prove that fn. $\boxed{f(x)=|x-1|}$, $x\in R$ is not Diff. at $\boxed{x=1}$.

Solⁿ:- RHD:- $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{|x+h-1|-|x-1|}{h} = \lim_{h\to 0} \frac{|\cancel{1}+h-\cancel{1}|-|1-1|}{h} = \lim_{h\to 0} \frac{\cancel{h}-0}{\cancel{h}} = \lim_{h\to 0}(1) = 1$

LHD $= \lim_{h\to 0} \frac{f(x-h)-f(x)}{-h} = \lim_{h\to 0} \frac{|x-h-1|-|x-1|}{-h} = \lim_{h\to 0} \frac{|1-h-1|-|1-1|}{-h} = \lim_{h\to 0} \frac{h-0}{-h}$

$= \lim_{h\to 0}(-1) = -1$

Here $\boxed{RHD \neq LHD}$ so not Diff. Ans//

Conti. & diff.

Q. $f(x) = |x| \rightarrow$ at $\boxed{x=0}$ check diff. ?

$\frac{1}{0} = \infty$

Q.49: Check $f(x) = [x]$, $0 < x < 3$ is diff. or not at $x = 1$ & $x = 2$. $[x] \rightarrow$ GIF.

Sol^n:- at $\boxed{x=1}$

RHD $\Rightarrow \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[x+h] - [x]}{h}$

$= \lim_{h \to 0} \frac{[1+h] - [1]}{h} \Rightarrow \lim_{h \to 0} \frac{1-1}{h} \Rightarrow \lim_{h \to 0} \frac{0}{h} = \underline{0}$

LHD $\Rightarrow \lim_{h \to 0} \frac{f(x-h) - f(x)}{-h} = \lim_{h \to 0} \frac{[x-h] - [x]}{-h}$

$= \lim_{h \to 0} \frac{[1-h] - [1]}{-h} = \lim_{h \to 0} \frac{0-1}{-h} \Rightarrow \lim_{h \to 0} \frac{\cancel{-}1}{\cancel{-}h}$

$= \lim_{h \to 0} \left(\frac{1}{h}\right) = \infty$

Here LHD is not define. So $f(x) = [x]$ is not diff at $x = 1$ Ans