Continuity & diff.

*Ex:- Show that func. $f(x) = x - [x]$ is disconti at all integers.

Sol^n:- $\because$ fun. is disconti. at all integers.

$\therefore$ let n be a integers.

So:- LHL $\neq$ RHL

$$\Rightarrow \lim_{x\to n^-} f(x) = \lim_{x\to n^+} f(x)$$

$$\Rightarrow \lim_{x\to n^-}(x-[x]) = \lim_{x\to n^+}(x-[x])$$

$$\Rightarrow \lim_{x\to n^-}(x) - \lim_{x\to n^-}[x] = \lim_{x\to n^+}(x) - \lim_{x\to n^+}[x]$$

$$\Rightarrow n-(n-1) = \cancel{n} - \cancel{n}$$

$$= 1 \neq 0$$

$$\Rightarrow 1 \neq 0 \Rightarrow \text{LHL} \neq \text{RHL}$$

So f is disconti at all integers.

Continuity & diff.

Ex:- Show that func. $f(x) = x - [x]$ is disContinuous at all integers.

Sol^n:- ∵ fun. is disConti. at all integers.

∴ let n be a integers.

So:- LHL ≠ RHL

$$\Rightarrow \lim_{x\to n^-} f(x) = \lim_{x\to n^+} f(x)$$

$$\Rightarrow \lim_{x\to n^-}(x-[x]) = \lim_{x\to n^+}(x-[x])$$

$$\Rightarrow \lim_{x\to n^-}(x) - \lim_{x\to n^-}[x] = \lim_{x\to n^+}(x) - \lim_{x\to n^+}[x]$$

$$\Rightarrow n-(n-1) = \cancel{n} - \cancel{n}$$

$$= 1 \neq 0$$

$\Rightarrow 1 \neq 0 \Rightarrow$ LHL ≠ RHL

So f is disConti at all integers.

Q. $f(x) = \begin{cases} x^2 \cdot \sin\frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$ is Conti. or not.

Sol^n if LHL = RHL = $f(0)$ then $f(x)$ is Conti.

$$\Rightarrow \text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left(x^2 \cdot \sin\frac{1}{x}\right) = 0$$

$$\left[\because -1 \leq \sin x \leq 1 \Rightarrow 1 \leq \sin(1/x) \leq 1 \right.$$

$$\Rightarrow -x^2 \leq x^2 \cdot \sin\left(\frac{1}{x}\right) \leq x^2$$

$$\Rightarrow \lim_{x\to 0^-}(-x^2) \leq \lim_{x\to 0^-}\left(x^2\cdot\sin\frac{1}{x}\right) \leq \lim_{x\to 0^-}(x^2)$$

$$\Rightarrow (-0) \leq \lim_{x\to 0^-}\left(x^2 \cdot \sin\frac{1}{x}\right) \leq (0)$$

$$\Rightarrow \left. \left[0 \leq \lim_{x\to 0^-}\left(x^2\cdot\sin\frac{1}{x}\right) \leq 0 \right] \right\}$$

RHL = 0

$f(0) = 0$

here LHL = RHL = $f(0)$ so $f(x)$ is Conti.

Continuity & diff.

Ex:- find value of K so f is Conti at indicated point.

$$f(x) = \begin{cases} \dfrac{K\cos x}{\pi - 2x} & ; x \neq \pi/2 \\ 3 & ; x = \pi/2 \end{cases} \quad \text{at } x = \pi/2$$

Sol^n:- $\because$ $f(x)$ is Conti then

$\Rightarrow [LHL = RHL = f(\pi/2)]$

So:- $\left[\lim_{x\to \pi/2^+} f(x) = f(\pi/2)\right]$

$\Rightarrow \lim_{x\to \pi/2^+} \left(\dfrac{K\cos x}{\pi - 2x}\right) = 3$

$\Rightarrow$ let $\left(x = \frac{\pi}{2} + h\right)$ then $h \to 0$ $\quad x \Rightarrow \pi/2$

$\Rightarrow \lim_{x\to \pi/2^+} \left\{\dfrac{K\cos(\pi/2 + h)}{\pi - 2(\pi/2 + h)}\right\} = 3$

$\Rightarrow \lim_{h\to 0} \left[\dfrac{K(-\sin h)}{\cancel{\pi} - \cancel{\pi} - 2h}\right] \Rightarrow \dfrac{K}{2} \lim_{h\to 0}\left(\dfrac{-\sin h}{-h}\right)$

$\Rightarrow \dfrac{K}{2} \boxed{\lim_{h\to 0}\left(\dfrac{\sin h}{h}\right)} \Rightarrow \dfrac{K}{2} = 3 \Rightarrow K = 6$ Ans

JEE21 Ex:- $f(x) = \begin{cases} 2\sin\left(\frac{-\pi x}{2}\right) & ; x < -1 \\ |ax^2 + x + b| & ; -1 \le x \le 1 \\ \sin(\pi x) & ; x > 1 \end{cases}$

is Conti then $(a+b) = ?$ -1 Ans

Sol^n:- $\because f(x) \to$ Conti $\Rightarrow$ LHL = RHL for $x = -1$ & $x = +1$

at $x = -1$

$\Rightarrow \lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$

$\Rightarrow \lim_{x\to -1^-} 2\sin\left(-\frac{\pi}{2}x\right) = \lim_{x\to -1^+} |ax^2 + x + b|$

$\Rightarrow 2\sin\left(-\frac{\pi}{2}(-1)\right) = |a - 1 + b|$

$\Rightarrow \boxed{2 = |a + b - 1|}$ — (1)

$\Rightarrow a + b - 1 = 2$ or $a + b - 1 = -2 \Rightarrow a + b = -1$

$a + b = 3$

at $x = +1$

$\lim_{x\to 1^-} |ax^2 + x + b| = \lim_{x\to 1^+} \sin(\pi x)$

$\Rightarrow |a + 1 + b| = \sin(\pi)$

$= \boxed{|a + b + 1| = 0}$ — (2)

$\Rightarrow a + b + 1 = 0 \Rightarrow a + b = -1$ Ans

ABLES® KOTA

Continuity & diff.

H.W.
Ex:- $f(x) = |x| - |x+1|$ find point of disconti.

let $f(x) = g(x) - h(x)$

where $g(x) = |x|$

↓

Conti

let $g(x) = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$

LHL = RHL = f(0)

↓

Cont

$h(x) = |x+1|$

↓

Conti

$x+1=0$

$x=-1$

$h(x) = \begin{cases} -(x+1) & ; x < -1 \\ (x+1) & ; x \geq -1 \end{cases}$

↓

LHL = RHL = f(−1)

↓

Conti.

if $f(x) = g(x) + h(x)$

such that $g(x) \rightarrow$ Conti

& $h(x) \rightarrow$ Conti

then $f(x) \rightarrow$ also Conti.

$\boxed{f(x) = g(x) - h(x)}$

$\boxed{f(x) = g(x) \cdot h(x)}$

$\boxed{f(x) = \frac{g(x)}{h(x)}}$