Continuity & Diff.

Ex:- (1) $f(x) = \begin{cases} \sin\frac{\pi}{2}x & ; x < 1 \\ [x] & ; x \geq 1 \end{cases}$ find weather f(x) is Conti or not at $\boxed{x=1}$

where [ ] → G.I.F.

Sol^n:- at $\boxed{x=1}$ fun is Conti if

$\Rightarrow \lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = f(1)$

$\Rightarrow \lim_{x\to 1^+} [x] = \lim_{x\to 1^-} \left(\sin\frac{\pi}{2}x\right) = [1]$

$\Rightarrow [1] = \sin\frac{\pi}{2}(1) = [1]$

$= \boxed{1 = 1 = 1} \Rightarrow$ Conti. Ans

Ex:- $f(x) = \begin{cases} \frac{|x|}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$ find point of dis.Continuity.?

Sol^n:- $f(x) = \begin{cases} \frac{x}{x} = 1 & ; x > 0 \\ \frac{-x}{x} = -1 & ; x < 0 \end{cases}$, 0 ; x = 0

Sol:- at $\underline{x=0}$ ⇒

LHL $\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-1) = -1$

RHL $= \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (1) = 1$

Now $f(0) = 0$

Here $\underline{LHL \neq RHL} \neq f(0)$

not Conti at $\boxed{x=0}$ Ans, so $x=0$ is the point of dis Continuity.

Continuity & Diff.

Ex: $f(x) = \begin{cases} x^{10}-1 & ; x \le 1 \\ x^2 & ; x > 1 \end{cases}$ find discon. point?

Sol^n:- let c be a real no. such that:-

i) $c < 1 \Rightarrow$ Conti if :- $f(c) = \lim\limits_{x\to c} f(x)$

$\Rightarrow c^{10}-1 = \lim\limits_{x\to c}(x^{10}-1)$

$= \left[c^{10}-1 = c^{10}-1\right]$ so:-

f is Conti at all values for $\boxed{c<1}$. Ans

ii) $c > 1 \Rightarrow$ Conti if $f(c) = \lim\limits_{x\to c} f(x)$

$\Rightarrow c^2 = \lim\limits_{x\to c}(x^2)$

$\Rightarrow c^2 = c^2$ so f is also Conti for all values of $c>1$.

iii) $\boxed{c=1} \to$ Conti if $f(c) = \lim\limits_{x\to c} f(x)$

$\Rightarrow$ or $f(c) = \lim\limits_{x\to c^+} f(x) = \lim\limits_{x\to c^-} f(x)$

$\Rightarrow c^{10}-1 = \lim\limits_{x\to 1^+}(x^2) = \lim\limits_{x\to 1^-}(x^{10}-1)$

$= (1)^{10}-1 = (1)^2 = (1^{10}-1)$

$= 0 \ne 1 \ne 0$ So at $\boxed{x=1}$ given func. is not Conti. Ans

Continuity & Diff.

Ex:- discuss Conti of fun.

$$f(x) = \begin{cases} -2 & ; x \leq -1 \\ 2x & ; -1 < x \leq 1 \\ 2 & ; x > 1 \end{cases}$$

Sol^n:- let c be a point on real no. line.

i) $c < -1 \rightarrow$ fun. is Conti if $f(c) = \lim_{x \to c} f(c)$

$\Rightarrow -2 = \lim_{x \to c} (-2) \Rightarrow \boxed{-2 = -2}$

So:- f is Conti $\rightarrow$ when $\boxed{c < -1}$

ii) $c = -1 \Rightarrow f(c) = \lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$

$\Rightarrow -2 = \lim_{x \to -1^+} (2x) = \lim_{x \to -1^-} (-2) \Rightarrow -2 = 2(-1) = -2$

$= \boxed{-2 = -2 = -2}$

So f is Conti. at $\boxed{c = -1}$ Ans

iii) $-1 < c < 1 \Rightarrow f(c) = \lim_{x \to c} f(x)$

$\Rightarrow 2c = \lim_{x \to c} (2x) \Rightarrow \boxed{2c = 2c}$

So:- fun. is Conti $c \in (-1, 1)$

iv) $\boxed{c = 1} \Rightarrow f(c) = \lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$

$\Rightarrow 2c = \lim_{x \to 1^+} (2) = \lim_{x \to 1^-} (2x) = 2(1) = 2 = 2(1)$

$\Rightarrow \boxed{2 = 2 = 2}$ so Conti at $\boxed{c = 1}$ Ans

v) $c > 1 \rightarrow f(c) = \lim_{x \to c} f(x)$

$\Rightarrow 2 = \lim_{x \to c} (2)$

$\Rightarrow \boxed{2 = 2} \rightarrow$ Conti at $c > 1$

So the given fun. is Conti at all real no. Ans

Continuity & Diff.

Ex. for what value of λ, $f(x) = \begin{cases} \lambda(x^2-2x) ; & x \le 0 \\ 4x+1 ; & x > 0 \end{cases}$ is Conti. at $\boxed{x=0}$ & what about Conti at $\boxed{x=1}$

H.W.

Sol^n:- $\because$ at $x=0$ func. is Conti if

$\Rightarrow f(0) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

$\Rightarrow \lambda(x^2-2x) = \lim_{x\to 0^+}(4x+1) = \lim_{x\to 0^-} \lambda(x^2-2x)$

$\Rightarrow \lambda(0-0) = 4(0)+1 = \lambda(0-2(0))$

$\Rightarrow \boxed{0 \ne 1 \ne 0}$

$[\text{RHL} \ne \text{LHL}]$

Here RHL & LHL not Concide with each other.

So there is no value of λ for which $f(x)$ is Conti at $x=0$.

Ex:- if $f(x) = \begin{cases} 3x-4 ; & 0 < x \le 2 \\ 2x + \underline{l} ; & 2 < x < 9 \end{cases}$

if f is Conti at $x=2$ then find value of l.