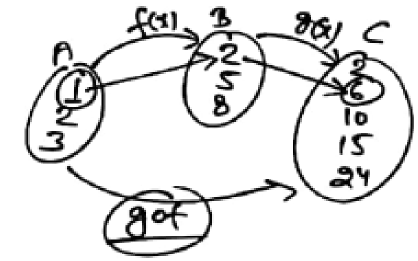
# Relation & function

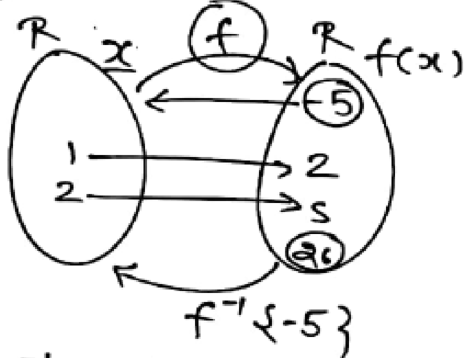



**Ex:-** $f : R \to R$ as a function:- $f(x) = x^2+1$ find.

i) $f^{-1}\{-5\}$ ii) $f^{-1}\{26\}$

$x^2+1 \to \cancel{<}0$

So:- $f^{-1}\{5\}$ is not possible. Ans




ii) $f^{-1}\{26\} = x \Rightarrow f(x) = 26$

$x^2+1 = 26 \Rightarrow x^2 = 25$

$\boxed{x = \pm 5}$

So:- $f^{-1}\{26\} = \pm 5$ Ans

So:- $f^{-1}\{-5\} = x$

$-5 = f(x)$

$\because f(x) = x^2+1$

So: $x^2+1 = -5$

$x^2 = -6 \Rightarrow x \Rightarrow$ Not Possible

**Ex:-** $f : A \to B$ & $g : B \to C$ as $\boxed{f(x) = 3x-1}$, $g(x) = 3x$

& $A = \{1,2,3\}$, $B = \{2,5,8\}$, $C = \{2,6,10,15,24\}$

find $gof(x) = ?$

Sol:- $gof(x) = g(f(x)) \Rightarrow g(f(1)) = g(2) = 6$

$\Rightarrow g(f(2)) = g(5) = 15$

$g(f(3)) = g(8) = 24$ Ans

# Relation & function

Ex: find inverse of $R \to R$

$$y = \frac{10^x + 10^{-x}}{10^x - 10^{-x}} = f(x)$$

$\Rightarrow$ Replace:- $x \leftrightarrow y$

$$\frac{10^y + 10^{-y}}{10^y - 10^{-y}}$$

$$\Rightarrow x = \frac{10^y + \frac{1}{10^y}}{10^y - \frac{1}{10^y}}$$

$$\Rightarrow x = \frac{\frac{10^{2y}+1}{10^y}}{\frac{10^{2y}-1}{10^y}} = \frac{10^{2y}+1}{10^{2y}-1}$$

$$\Rightarrow x = \frac{10^{2y}+1}{10^{2y}-1} \Rightarrow \text{let } \boxed{10^{2y} = t}$$

$$x = \frac{t+1}{t-1} \Rightarrow xt - x = t+1$$

$$\Rightarrow xt - t = x+1$$

$$\Rightarrow t(x-1) = x+1$$

$$\Rightarrow t = \frac{x+1}{x-1}$$

$$\Rightarrow \boxed{10^{2y} = \frac{x+1}{x-1}}$$

take log:- with base 10-

$$\Rightarrow \log_{10} 10^{2y} = \log_{10}\left(\frac{x+1}{x-1}\right)$$

$$\Rightarrow 2y = \log_{10}\left(\frac{x+1}{x-1}\right)$$

$$\boxed{y = \frac{1}{2}\log_{10}\left(\frac{x+1}{x-1}\right)} = f^{-1}(x)$$

Ans

Ex: If $f:[1,2] \to [5,8]$ as $\boxed{f(x) = mx+c}$ then find value of $\boxed{m \,\&\, c}$ such that $f(x)$ is invertible.

Solⁿ:- $\because f(x)$ is invertible when $f(x)$ is bijective:- so

① $f(1) = 5$ or ② $f(1) = 8$

$f(2) = 8$ $\quad$ $f(2) = 5$

Now:- $f(1) = 5 \Rightarrow m(1)+c$

$\Rightarrow \boxed{m + c = 5}$ — ①

$f(2) = \boxed{8 \Rightarrow 2m + c}$ — ②

$\Rightarrow 8 = 2m + (5-m)$

$\Rightarrow 8 = m+5 \Rightarrow m = 3$

$\Rightarrow c = 2$

Ans

# Relation & function

**Ex:** if $f(1)=1,\ n\geq 1$

$f(n+1)=2f(n)+1$ then $\boxed{f(n)=?}$

A) $2^{n+1}$ (B) $2^{n}$ (C) $2^{n}-1$ (D) $2^{n-1}-1$

**Solⁿ:-** $\boxed{f(1)=1}$

$\boxed{n=1} \Rightarrow f(1+1)=f(2)=2f(1)+1=2(1)+1=\boxed{3=f(2)}$

$\boxed{n=2} \Rightarrow f(2+1)=f(3)=2f(2)+1=2(3)+1=\boxed{7=f(3)}$

$\boxed{n=3} \Rightarrow f(3+1)=f(4)=2f(3)+1=2(7)+1=\boxed{15=f(4)}$

then $\boxed{f(n)=2^{n}-1}$ Ans

**Ex:-** if $\left[2f(x)+3f\left(\frac{1}{x}\right)=x-1\right]$ — (1), $\forall x\neq 0$ find $f(x)=?$

**Solⁿ:-** let Replace:- $x\to\frac{1}{x} \Rightarrow 2f\left(\frac{1}{x}\right)+3f(x)=\frac{1}{x}-1$ — (2)

$\Rightarrow$ let $\boxed{f(x)=p}$, $f\left(\frac{1}{x}\right)=q \Rightarrow 2p+3q=x-1$ — (3) $\times 2$

$3p+2q=\frac{1}{x}-1$ — (4) $\times 3$

$\Rightarrow 4p+6q=2x-2$

$9p+6q=\frac{3}{x}-3$

$-5p=2x-\frac{3}{x}+1$

$-5p=\frac{2x^2-3+x}{x}$

$p=\frac{-1}{5x}(2x^2-3+x)$

So $\boxed{f(x)=-\frac{2}{5}x+\frac{3}{5x}-\frac{1}{5}}$ Ans

**Ex:-** $f(0)=1$ & $f(1)=2$

$f(x)=\frac{1}{2}\left[f(x+1)+f(x+2)\right]$

find $\boxed{f(5)=?}=+12$ Ans

**Solⁿ:-** $2f(x)=f(x+1)+f(x+2)$

$\Rightarrow f(x+2)=2f(x)-f(x+1)$

$x=0 \Rightarrow f(2)=2f(0)-f(1)=2-2=0$

$x=1 \Rightarrow f(3)=2f(1)-f(2)=4-0=4$

$x=2 \Rightarrow f(4)=2f(2)-f(3)=0-4=-4$

$x=3 \Rightarrow f(5)=2f(3)-f(4)=8-(-4)=12$ Ans