

Relation & function

Ex:- \wedge on the set $A = \{1, 2, 3, 4, 5\}$
 defined by:- $(a \wedge b) = \min\{a, b\}$
 write the operation table of (\wedge) .

\wedge	①	2	3	4	5
①	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Ex:- $*$ on set $\{1, 2, 3, 4, 5\}$

$a * b = \text{HCF of } a \& b.$

$1 * 1 = 1$

$1 * 2 = 1$

$1 * 3 = 1$

$2 * 1 = 1$

$2 * 2 = 2$

$2 * 3 = 1$

$2 * 4 = 2$

$2 * 5 = 1$

$2 * 6 = 2$

$2 * 1 \quad 2 * 3 * 1$

$2 * 1 = 2$

- i) Compute:- $(2 \wedge 3) \wedge 4$ & $2 \wedge (3 \wedge 4)$
- ii) $(2 \wedge 3) \wedge (4 \wedge 5) = 2 \wedge 4 = 2$

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- e.g.:- $*$ on $N \rightarrow a * b = \text{LCM of } a \& b$
- i) $5 * 7 \rightarrow 5 * 7 = \text{LCM of } 5 \& 7 = 35$
 - ii) $20 * 16 \rightarrow 20 * 16 = \text{LCM of } 20 \& 16 = 80$
 - iii) Comm. ? ✓
 - iv) identity element of $*$ in N .

$\Rightarrow a * e = a$

$\text{LCM of } \begin{matrix} a \\ \downarrow \\ 20 \end{matrix} \& \begin{matrix} e \\ \downarrow \\ 1 \end{matrix} = 20$

$*$ $\rightarrow \{1, 2, 3, 4, 5\}, a * b = \text{LCM of } a \& b$

$*$	1	2	3	4	5
1			3	4	5
2			6		
3			3		
4					
5				20	5

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Ex! - $a * b = a^2 + b^2$ ✓

→ $(a * b) * c = (a^2 + b^2) * c$

$(a^2 + b^2)^2 + c^2$ ←
 $256 \leftarrow \frac{16}{4} \frac{16}{4} + 1 = 257$

→ $a * (b * c) = a * (b^2 + c^2)$

$= \frac{a^2}{4} + \frac{(b^2 + c^2)^2}{(4+1)^2}$ ←

$4 + 25 = 29$

Ex! $\frac{a * b}{4} = \frac{ab}{4} \leftarrow * c = \frac{ab}{4} * c = \frac{abc}{4}$ ✓

$b * a = \frac{ba}{4}$ ✓

$b * c = \frac{bc}{4} \Rightarrow a * \frac{bc}{4} = \frac{a * bc}{4} = \frac{abc}{4}$ ✓

$\frac{a * bc}{4} \leftarrow \frac{N}{D}$

$\Rightarrow \frac{abc}{4} \times \frac{1}{4} = \frac{abc}{16}$ ✓

Ex! - $a * b = ab^2$

Comm.
 asso →

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Ex:- Let $f(x) = x^2$, $g(x) = 2^x$
 then the solⁿ set of $[f \circ g(x) = g \circ f(x)]$ is

A) \mathbb{R} (B) $\{0\}$ (C) $\{0, 2\}$ (D) none

Solⁿ:- $f(x) = x^2$; $g(x) = 2^x$

$\therefore f \circ g(x) = f[g(x)] = f[2^x] = (2^x)^2$

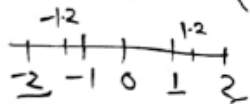
$\checkmark f \circ g(x) = 2^{2x}$

$\rightarrow g \circ f(x) = g[f(x)] = g[x^2] = 2^{x^2}$

So: $2^{2x} = 2^{x^2} \Rightarrow 2x = x^2 \Rightarrow x^2 - 2x = 0$

$x=0$ $x=2$

Solⁿ:- $\{0, 2\}$



$[x] = [1.0] = 1$	
$[1.5] = -2$	$[1.2] = 1$
$[1.9] = -2$	$[-1.2] = -2$
$[-1.0] = -1$	

Q. $f(x) = \begin{cases} -1 & ; x < 0 \\ 0 & ; x = 0 \\ 1 & ; x > 0 \end{cases}$

$g(x) = 1 + x - [x] \rightarrow$ greatest int. fun.
 Find $f[g(x)] = ?$

A) x (B) 1 (C) $f(x)$ (D) $g(x)$

Solⁿ:- $f[g(x)]$

$f[1 + x - [x]]$

Here $1 + x - [x]$ is always true.

So: $f(\text{true}) \Rightarrow f(x > 0) = 1$

$1 + x - [x] \Rightarrow x = 1.5 \Rightarrow 1 + 1.5 - [1.5] = 2.5 - 1 = 1.5$
 $x = -1.5 \Rightarrow 1 + (-1.5) - [-1.5] = 1 - 1.5 - (-2) = 1 - 1.5 + 2 = 3 - 1.5 = 1.5$