Relation & function

Defination:- a given Binary operation:- $*: A\times A \to A$, if there is a Identity element $e \in A$, for any element $\boxed{a \in A}$, if there exist another element $b \in A$ such that:- $[a*b = b*a = e]$

then a is invertible and inverse of element a is denoted as $a^{-1} = b$

$a^{-1} = b$

Ex:- Show that:- $(-a)$ is the inverse of a additive operation in R. & $\frac{1}{a}$ is $(a \neq 0)$ the inverse of multiplication operation in R.

Soln:- ∵ for additive operation in R the identity element $\boxed{e = 0}$

∴ for $a \in R$ there exist $-a \in R$ such that:- $a*b = a+b$

So:- $a*(-a) = a+(-a) = a-a = 0$

or $(-a)*a = -a+a = 0$

So:- $[a*(-a) = -a*a = 0]$

So:- $-a$ is inverse of a

Relation & function

Sol^n:- ∵ for multiplication operation on R there is $\boxed{e=1}$.

so:- for $a \in R \to$ there exist $\frac{1}{a} \in R$ such that. $a * b = a \cdot b$

i.e. ⇒ $\left[a * \frac{1}{a} = a \cdot \frac{1}{a} = 1 = \frac{1}{a} * a\right]$ so:- $\frac{1}{a}$ is inverse of a. Ay

Ex:- Check $*$ is binary operation or not.

i) on Z (non neg.) ⇒ $a * b = a - b$ $\{0, 1, 2, 3\}$

∵ Here for $(2,1) \to$ $2 * 1 = 2 - 1 = 1 \in Z^{\text{nonzero}}$

but for $(1,2) \to$ $1 * 2 = 1 - 2 = -1 \notin Z^{\text{nonzero}}$

means there is no image of pair $(1,2)$ in Z^+ so it is not a funct. and not a B.O.

ii) on $R \to * \to a * b = ab^2$

∵ for the given operation:. $a * b = ab^2$ there is always a image of pair (a, b) in R.

So it is B.O.

iii) on $Z^+ \to$ $a * b = |a - b|$ ⇒ $Z^+ = \{1, 2, 3, 4 \, ---\}$

∵ for any pair of $(a,b) \to a * b = |a - b| > 0 \in Z^+$ so:- it is a fun. & a B.O.

Relation & function

Sol^n :- ∵ for multiplication operation on R there is $\boxed{e=1}$.

so:- for $a \in R \rightarrow$ there exist $\frac{1}{a} \in R$ such that. $a * b = a \cdot b$

i.e. $\Rightarrow \left[a * \frac{1}{a} = a \cdot \frac{1}{a} = 1 = \frac{1}{a} * a \right]$ so:- $\frac{1}{a}$ is inverse of a.

Ex:- Check $*$ is binary operation or not.

i) on $Z^{\text{non neg.}} \Rightarrow a * b = a - b$ $\{0,1,2,3\}$

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ii) on $R \rightarrow * \rightarrow a * b = ab^2$

∵ for the given operation;

$a * b = ab^2$ there is always a image of pair (a,b) in R

so it is B.O.

iii) on $Z^+ \rightarrow a * b = |a-b| \Rightarrow Z^+ = \{1,2,3,4 ---\}$

∵ for any pair of $(a,b) \rightarrow a * b = |a-b| \geq 0 \notin Z^+$ so:. it is not a fun.

Relation & function

Ex:- which * is comm. & asso.

i) on $Z^+ \rightarrow a*b = 2^{ab}$

sol^n:- $\because a*b = 2^{ab} \in Z^+$

so:- $b*a = 2^{ba} \in Z^+$

Here! $[a*b = b*a] = [2^{ab} = 2^{ba}]$ ✓

$a=2, b=2, c=$

so it is comm.

$(a*b)*c = (2^{ab})*c = 2^{(2^{ab}\cdot c)} = 2^{(2^4 \cdot 1)} = 2^{16}$

& $a*(b*c) = a*(2^{bc}) = (2)^{a\cdot 2^{bc}} = 2^{2\cdot 2^2} = 2^8$

Here $(a*b)*c \neq a*(b*c)$

So not associative.

ii) on $Q \rightarrow a*b = ab+1$

Ration no. $= p/q$ $(q \neq 0)$

Comm. $\rightarrow \because a*b = ab+1$

& $b*a = ba+1$

Here! $[a*b = b*a]$ ✓

$\rightarrow$ asso.

$\because (a*b)*c = (ab+1)*c$

$= (ab+1)c+1 = abc+c+1$

& $a*(b*c) = a*(bc+1)$

$= a(bc+1)+1$

$= abc+a+1$

Here! $(a*b)*c \neq (a)*(b*c)$

So not asso.

iii) $R-\{1\} \Rightarrow a*b = \frac{a}{b+1}$

iv) $Q \rightarrow a*b = \frac{ab}{2}$