#Relation & function

Ex:- show [$\wedge : R \times R \to R$] given by $(a,b) \to \max(a,b)$

& $\vee : R \times R \to R$ given by $(a,b) \to \min(a,b)$

these are Binary operation.

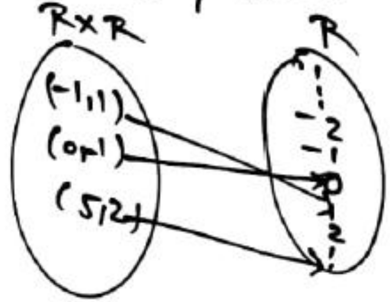
Sol^n:- $\because$ $\wedge : R \times R \to R$ given by $(a,b) \to \max(a,b)$

is a function

because:- let $a = -1,\ b = 1 \to (-1,1) \Rightarrow \wedge(-1,1) \Rightarrow 1$

$a = 0,\ b = -1 \to (0,-1) \to \wedge(0,-1) \to 0$

$a = 5,\ b = 2 \to (5,2) \to \wedge(5,2) \to 5$



then $\wedge$ is a Binary operation. Ans

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Defination:

a Binary operation:- $*$ on the set A is called Commutative if $a*b = b*a \quad \forall \; \boxed{a,b \in A}$

Ex:- Show $+ : R\times R \to R$ & $\times : R\times R \to R$ is Commutative.

Sol^n:- $+ : R\times R \to R$ is given by $(a,b) = a+b$

So:- $+(a,b) \Rightarrow a+b$ & Now $+(b,a) = b+a$

here it is clear $[a+b = b+a]$ means $a*b = b*a$

So $+$ is Commutative. Ans

Now:- $\times : R\times R \to R$ is given by:- $(a,b) = a\times b$

Now:- $\boxed{a\times b = b\times a}$ means $(a*b) = (b*a)$ for $\forall$ $a,b,\in R$

So $\times$ is also Comm.

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Ex: Show :- $* : R \times R \to R$ given by $(a, b) = a + 3b$ is not comm.

Sol^n :- $\because$ $* \to (a,b) = a+3b$

$* \to (b,a) = b+3a$

let :- $a = 5,\ b = 2 \to (a,b) = 5 + 3(2) = 11$

$\hookrightarrow (b,a) = 2 + 3(5) = 17$

Here $(a,b) \neq (b,a)$

$a+3b \neq b+3a$

$\boxed{11 \neq 17}$ So not comm.

Definition :- a Binary operation $* : A \times A \to A$ is said to be associative if

$$(a*b)*c = a*(b*c),\ \forall a, b, c \in A$$

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Ex: Check $* : R \times R \to R$ given by $\boxed{(a, b) = (a + b^2)}$ is asso. or not?

Sol^n:- $\because * \to (a, b) = (a + b^2)$

$\because$ for associative :- $(a * b) * c = \{(a + b^2) * c\} = (a + b^2) + c^2 = \text{LHS}$

Now:- $a * (b * c) = a * (b + c^2) = a + (b + c^2)^2 = \text{RHS}$

Here $\text{LHS} \neq \text{RHS}$ because :- let $a = 2, b = 3, c = 1 \in R$

$\text{LHS} = 2 + (3)^2 + (1)^2 = 12$

$\text{RHS} = 2 + (3+1)^2 = 2 + (4)^2 = 18 \Rightarrow \boxed{12 \neq 18}$ so not asso.

Definition:- a Binary operation $* : A \times A \to A$ is said to be associative if

$$(a * b) * c = a * (b * c), \quad \forall a, b, c \in A$$

$\text{LHS} = a + b^2 + c^2$ | $\text{RHS} = a + b^2 + c^4 + 2bc^2$

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Defination a Binary operation $*: A \times A \to A$ if an element $\boxed{e \in A}$ is such that:-

$$a * e = e * a = a \quad \forall \boxed{a \in A}$$

then $\boxed{e \text{ is called Identity Element}}$

Ex:- Show zero is a Identity Element for addition on R.

Soln:- ∵ zero is an Identity element means

$$\boxed{e = 0} \Rightarrow \boxed{a * e = a + e = a + 0 = a = e + a}$$

So it is Clear that zero is Identity for addition on R.

Ex:- Check zero is a Identity Element for add. on N.

Soln:- ∵ zero doesn't belong to N i.e. $0 \notin N$

So zero is not Identity element for add on N.

Ex:- Show one is a Identity Element for multiplication on R:-

let $e = 1$ for multi → $\times \to (a, e) \Rightarrow a \times e = a \times 1 = a$ so it is.