Relation & function

Ques:- $f: R \to R$ as $f(x) = |x| \to$ one-one / many & onto / into.

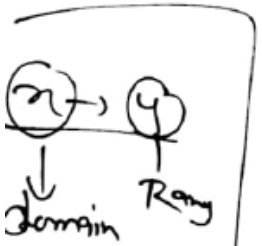
Soln:- $\because f(x) = |x| \longrightarrow f(x) = \begin{cases} x \to x \geq 0 \\ -x \to x < 0 \end{cases}$

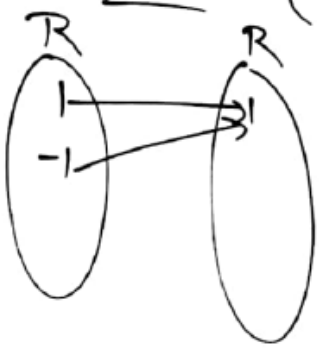
$\Rightarrow$ let $x = 1 \to f(x) = x = 1 \Rightarrow f(x) = 1$

& $x = -1 \to f(x) = -x = -(-1) = 1 \Rightarrow f(x) = 1$

So!- Here!- $f(-1) = f(1) = 1$ but $(1) \neq (-1)$

So it is many one

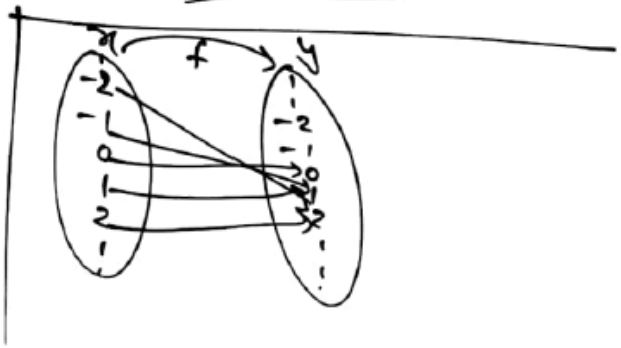




$f(x_1) = f(x_2)$

$\downarrow$

$x_1 = x_2$



let $x = -2 \in R$

Here for the value $x = -2$ there is an image in y. such that $y = 2 \in R$

But Now Here we can see for $y = -2 \in R$ there doesn't exist any pre image in x. So it is into function.

Relation & function

Ques: $f: R \to R$

$$f(x) = \begin{cases} 1 & ; x > 0 \\ 0 & ; x = 0 \\ -1 & ; x < 0 \end{cases}$$

→ Signum function

↓

never one-one nor → onto

(1) Here from the given function it is clear that the value of $f(x)$ is always restrict upto 3 values i.e. $1, 0$ & -1 so it is many-one.

or (2) let: $x = 1 \to f(1) = 1$, $x = 5 \to f(5) = 1$] Here

$f(1) = f(5) = 1$

but $1 \neq 5$ many-one

∵ Here for $y \in R$ there is only 3 value which has pre-image in x. i.e. only $-1, 0, 1$ has pre-image in x.

So:- for $y = 5 \in R$ there is no-preimage in x.

Hence into. Ans.

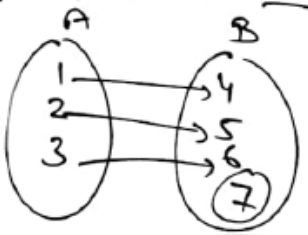
$f: R \to R$

$$f(x) = \begin{cases} 5 & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

Relation & function

Que $A = \{1,2,3\}$, $B = \{4,5,6,7\}$

$f = \{(1,4)(2,5)(3,6)\} \rightarrow A \rightarrow B$



Ques: $f: R \rightarrow R \rightarrow f(x) = 1 + x^2$

$\Rightarrow$ let $f(x_1) = f(x_2)$

$1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$x_1 = x_2$ $\quad$ $x_1 = -x_2$

onto:- Here $f(x) = 1 + x^2$ (is always $\rightarrow$ +ive)

it means for -ive value of $y \in R$ there is no pre-image in $x \rightarrow$ into.

Ques:- $f: N \rightarrow N$; $f(n) = \begin{cases} \frac{n+1}{2} & ; n \rightarrow \text{odd} \\ \frac{n}{2} & ; n \rightarrow \text{even} \end{cases}$

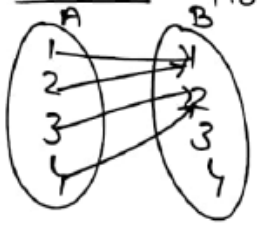
Soln.

one-one $\rightarrow$ let $\Rightarrow n = 1 \rightarrow f(n) = \frac{1+1}{2} = 1 = f(1)$

$n = 2 \rightarrow f(2) = \frac{2}{2} = 1 \Rightarrow f(2) = 1$

$\therefore f(1) = f(2)$ but $1 \neq 2$ so many-one.

$\Rightarrow$ onto:-



Here $n = 1 \Rightarrow f(1) = 1$

$n = 2 \Rightarrow f(2) = 1$

$n = 3 \Rightarrow f(3) = 2$

$n = 4 \Rightarrow f(4) = 2$

so here for $y \in N$ there is always a pre-image in x.

onto

Relation & function

Ques. If $A = R - \{3\}$, $B = R - \{1\}$, $f: A \to B \to f(x) = \frac{x-2}{x-3}$

Soln:- let for x_1, x_2 →

$f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 + 2x_1 = -3x_2 + 2x_2$

$\Rightarrow -x_1 = -x_2 \Rightarrow \boxed{x_1 = x_2}$

Some-one it is :-

So it is one-one ✓

onto :- the function $f(x)$ is onto.

if $\boxed{f(x) = y}$

$\because f(x) = \frac{x-2}{x-3} \Rightarrow y = \frac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$xy - 3y = x - 2$

$\Rightarrow xy - x = 3y - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow \boxed{x = \frac{3y-2}{y-1}}$ where $\boxed{y \neq 1}$

So here for any value of y there exist x such that $\boxed{f(x) = y}$

So :- $f\left(\frac{3y-2}{y-1}\right) = \frac{\left(\frac{3y-2}{y-1}\right)-2}{\frac{3y-2}{y-1}-3} = \frac{3y-2-2y+2}{3y-2-3y+3}$

$\Rightarrow f(x) = \frac{y}{1} \Rightarrow \boxed{f(x) = y}$ ✓ onto