

# Relation & function #

Ex: is  $f: \mathbb{R} \rightarrow \mathbb{R}$  as  $f(x) = x^2$

check it one-one / many-one and onto or into.

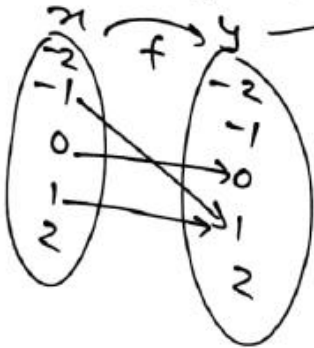
Sol<sup>n</sup>:- let  $f(x_1) = f(x_2)$

$$\Rightarrow x_1^2 = x_2^2$$

$$\Rightarrow x_1 = \pm x_2$$

$$x_1 = x_2 \quad \& \quad (x_1 = -x_2)$$

So it is many-one. ✓



$$\therefore f(x) = x^2 \Rightarrow y = x^2$$

$$\text{So: } x = \pm \sqrt{y}$$

$$\text{let } y = 2 \Rightarrow x = \sqrt{2} \in \mathbb{R}$$

$$\text{but } y = -2 \Rightarrow x = \pm \sqrt{-2} \notin \mathbb{R}$$

So it is into. ✓✓

# Relation & function #

Ex:  $f: \mathbb{R}_* \rightarrow \mathbb{R}_*$ ;  $f(x) = \frac{1}{x}$  → Check it one-one / many-one and onto or into.

$\mathbb{R}_* \rightarrow$  set of all non-zero real no.

Sol<sup>n</sup>:- if  $f(x_1) = f(x_2)$

$$\frac{1}{x_1} = \frac{1}{x_2}$$

$$\Rightarrow \boxed{x_1 = x_2}$$

So it is one-one.

# onto:-  $f(x) = \frac{1}{x} \Rightarrow \boxed{y = \frac{1}{x}}$

$$\Rightarrow \boxed{x = \frac{1}{y}}$$

$\rightarrow y \in \mathbb{Q} \rightarrow \boxed{x = \frac{1}{y}} \in \mathbb{R}_*$  so it is onto

$\rightarrow y = 1 \rightarrow \boxed{x = 1} \in \mathbb{R}_*$

$$\underline{f(x) = \frac{1}{x} \Rightarrow y = \frac{1}{x} \Rightarrow \boxed{x = \frac{1}{y}}}$$

$$\Rightarrow f\left(\frac{1}{y}\right) = \frac{1}{\frac{1}{y}} = y$$

$\Rightarrow \boxed{f\left(\frac{1}{y}\right) = y} \rightarrow$  it is onto.

ques: ii)  $f: \mathbb{N} \rightarrow \mathbb{N}$ ;  $f(x) = x^3$  # Relation & function #

sol<sup>n</sup>:  $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2 \rightarrow$  one-one

#  $\therefore f(x) = x^3 \Rightarrow y = x^3 \Rightarrow x = (y)^{1/3}$

Here for values of  $y$ ,  $x$  doesn't belong to  $\mathbb{N}$ .  
So it is into function.

Ques:  $f: \mathbb{R} \rightarrow \mathbb{R}$ ;  $f(x) = [x]$

where  $[x] \rightarrow$  greatest integer less than or equal to  $x$ .

$\Rightarrow$  one-one:  $f(x) = [x]$

Let  $x = 1.8 \Rightarrow f(1.8) = [1.8] = 1$

&  $x = 1.2 \Rightarrow f(1.2) = [1.2] = 1$

So it is many-one.

ii)  $f: \mathbb{Z} \rightarrow \mathbb{Z}$ ;  $f(x) = x^3$

one-one  $\rightarrow$  ✓

$\Rightarrow y = x^3 \Rightarrow x = (y)^{1/3}$

Here if  $y = 2 \rightarrow x = (2)^{1/3} \notin \mathbb{Z}$   
So it is into.

iii)  $f: \mathbb{R} \rightarrow \mathbb{R}$ ;  $f(x) = x^3$

onto:  $f(x) = [x]$

if  $f(x) = 0.7$  then:-

$0.7 \neq [x]$

So it is into function.