Class - Test → 2

Ex. (i) $A = \{5, 6, 7, 9\}$

$R = \{(5,5)(6,6)(7,7)(9,7)\}$

$\{(0,0)(1,1)(2,2) \ldots (12,12)$
$(4,0)(0,4)(5,1)(1,5)(2,6)(6,2)$
$\ldots (8,12)(12,8)$
$\ldots \}$ Ans.

i) Ref. → $(5,5) \in R$, $(6,6) \in R$, $(7,7) \in R$, $(9,9) \in R$ — 1

So:- to make R → Reflexive we need to add $\boxed{(9,9) \in R}$

ii) Symm. → $(9,7) \in R \Rightarrow (7,9) \in R$

So to make Relation symm. we need to add $\boxed{(7,9) \in R}$

So:- Ans → $(9,9)$ & $(7,9)$ Ans

ii) $A = \{0, 1, 2 \ldots 11, 12\}$ & $R = \{(a,b);\ |a-b|$ is multiple of 4.

i) Ref. → ∵ $|a-b|$ is multiple of 4 ⇒ $|a-a| = 0$ is multiple of 4. so it is ref.

ii) Symm → ∵ $|a-b|$ is multiple of 4 ⇒ $|b-a|$ is also multiple of 4 → So Symm

iii) Trans → $|a-b|$ is multiple of 4 & $|b-c|$ is also multiple of 4 ⇒ $|a-c|$ should be multiple of 4 → so Trans.

} equi.

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(1) $A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \to \underline{A^n} = ?$

$$A^2 = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 9^2 & 0 & 0 \\ 0 & 9^2 & 0 \\ 0 & 0 & 9^2 \end{bmatrix} \Rightarrow A^n = \begin{bmatrix} 9^n & 0 & 0 \\ 0 & 9^n & 0 \\ 0 & 0 & 9^n \end{bmatrix} = C \text{ Ans}$$

(2) $a_{ij} = \dfrac{(i+2j)^2}{2} \Rightarrow A = [a_{ij}]_{2\times 2}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 9/2 & 25/2 \\ 8 & \underline{18} \end{bmatrix} \to \text{(B) Ans}$$

(3) $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & -2 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & x \end{bmatrix}, C = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, D = \begin{bmatrix} 15+x \\ 1 \end{bmatrix}$

given

$\Rightarrow (2A-3B)C = D \Rightarrow x = ?$

$$\Rightarrow \begin{bmatrix} 8 & 2 & 0 \\ 2 & -4 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 0 & -3 \\ 9 & 3 & 3x \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 2 & 3 \\ -7 & -7 & 4-3x \end{bmatrix}_{2\times 3}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 9 \\ -21+4-3x \end{bmatrix} = \begin{bmatrix} 15+x \\ 1 \end{bmatrix} \Rightarrow$$

$9 = 15 + x$

$x = -6$ Ans

$-17 - 3x = 1$

$-18 = 3x$

$\Rightarrow x = -6$ Ans

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(6) $A^2 - A + I = 0 \Rightarrow A^{-1}$

$\Rightarrow A^{-1} \cdot A^2 - A^{-1} \cdot A + A^{-1} \cdot I = A^{-1} \cdot 0 \Rightarrow A - I + A^{-1} = 0 \Rightarrow \boxed{A^{-1} = I - A}$ ✓ (A)

(7) if $f(x) = x^2 + 4x - 5$ & $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, $f(A) = ?$

$\Rightarrow f(A) = A^2 + 4A - 5I$

(8) $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$

A) $A \cdot A^T = I$

B) $B \cdot B^T = I$

C) $AB \neq BA$

D) $(AB)^T = I$ ✓

$\Rightarrow AB = \begin{bmatrix} 8-7 & 2-2 \\ -28+28 & -7+8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\downarrow$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q. 101 $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ # (Lesg - Test → 2 #

then $(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

A) $I + A$ C) $A - I$

B) $I - A$ D) A

$\cos 2\alpha = 2\cos^2\alpha - 1 \;|\; 1 - 2\sin^2\alpha$

$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$

$\tan 2\alpha = \sin 2\alpha / \cos 2\alpha$

101. $(I-A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$

01. $\begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\begin{bmatrix} \cos\alpha + \sin\alpha\tan\alpha/2 & -\sin\alpha + \cos\alpha \cdot \tan\alpha/2 \\ -\cos\alpha \cdot \tan\alpha/2 + \sin\alpha & \sin\alpha \cdot \tan\alpha/2 + \cos\alpha \end{bmatrix}$

$\begin{bmatrix} 2\cos^2\alpha/2 - 1 + 2\sin\alpha/2\cos\alpha/2 \cdot \tan\alpha/2 & -2\sin\alpha/2\cos\alpha/2 + (2\cos^2\alpha/2 - 1)\cdot\tan\alpha/2 \\ -(2\cos^2\alpha/2 - 1)\cdot\tan\alpha/2 + 2\sin\alpha/2\cos\alpha/2 & 2\sin\alpha/2\cos\alpha/2\tan\alpha/2 + 2\cos^2\alpha/2 - 1 \end{bmatrix}$

$-2\sin\alpha/2\cos\alpha/2 + 2\cos^2\alpha/2 \cdot \tan\alpha/2 - \tan\alpha/2$

$\begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix}$

$-\tan\alpha/2 \cdot 2\cos^2\alpha/2 + \tan\alpha/2 + 2\cos\alpha/2\sin\alpha/2$

$\begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$

$I + A$

A