Determinant

Cremer Rule:-

Let:- $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

Here:- we can find x, y, z as

$\Rightarrow x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$

✓ $\left[x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}, \quad z = \frac{\Delta_z}{\Delta} \right]$

Here $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

✓

$\Delta_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$\Delta_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$

$\Delta_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_2 & d_3 \end{vmatrix}$

Determinant

$\boxed{1+1+2=4}$

Ex:-

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4 \Rightarrow 2u + 3v + 10w = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \Rightarrow 4u - 6v + 5w = 1$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \Rightarrow 6u + 9v - 20w = 2$

Let $\frac{1}{x} = u$, $\frac{1}{y} = v$, $\frac{1}{z} = w$

By Cramer Rule :-

$$\Delta = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix} = 2(120-45) - 3(-80-30) + 10(36+36)$$
$$= 150 + 330 + 720 = 1200$$

$$\Delta_u = \begin{vmatrix} 4 & 3 & 10 \\ 1 & -6 & 5 \\ 2 & 9 & -20 \end{vmatrix} = 4(+120-45) - 3(-20-10) + 10(9+12)$$
$$= 300 + 90 + 210 = 600$$

$288 - 48 = 240$ $\quad$ $72 \times 4 = 288$

$$\Delta_v = \begin{vmatrix} 2 & 4 & 10 \\ 4 & 1 & 5 \\ 6 & 2 & -20 \end{vmatrix} = 2(-30) - 4(-110) + 10(8-6)$$
$$= -60 + 440 + 20 = 400$$

$$\Delta_w = \begin{vmatrix} 2 & 3 & 4 \\ 4 & -6 & 1 \\ 6 & 9 & 2 \end{vmatrix} = 2(-21) - 3(2) + 4(36+36)$$
$$= -42 - 6 + 288$$
$$= 240$$

Now by Cramer Formula :-

$u = \frac{\Delta_u}{\Delta} = \frac{600}{1200} = \frac{1}{2} \Rightarrow \boxed{u = \frac{1}{2}} \Rightarrow \frac{1}{x} = \frac{1}{2} \Rightarrow x=2$

$v = \frac{\Delta_v}{\Delta} = \frac{400}{1200} = \frac{1}{3} \Rightarrow \boxed{v = \frac{1}{3}} \Rightarrow \frac{1}{y} = \frac{1}{3} \Rightarrow y = 3$

$w = \frac{\Delta_w}{\Delta} = \frac{240}{1200} = \frac{1}{5} \Rightarrow \boxed{w = \frac{1}{5}} \Rightarrow \frac{1}{z} = \frac{1}{5} \Rightarrow z=5$

Determinant

Ex:- $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = \boxed{2xyz(x+y+z)^3}$ $\left(\because a^2-b^2 = (a-b)(a+b)\right)$ $\quad \cancel{x}-y-\cancel{z}-\cancel{x}-y+\cancel{z}$

$$\Delta = \frac{xyz}{xyz}\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix}$$

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x(y+z)^2 & x^2y & zx^2 \\ xy^2 & (x+z)^2y & y^2z \\ xz^2 & yz^2 & (x+y)^2z \end{vmatrix}$$

$$\Delta = \frac{\cancel{xyz}}{\cancel{xyz}}\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$$

$\Rightarrow \quad C_2 \to C_2 - C_1, \quad C_3 \to C_3 - C_1$

$$\Delta = \begin{vmatrix} (y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2 \\ y^2 & (x+z)^2-y^2 & 0 \\ z^2 & 0 & (x+y)^2-z^2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} (y+z)^2 & [x-(y+z)](x+y+z) & [x-(y+z)](x+y+z) \\ y^2 & (x+z-y)(x+z+y) & 0 \\ z^2 & 0 & (x+y-z)(x+y+z) \end{vmatrix}$$

$$\Delta = (x+y+z)^2\begin{vmatrix} (y+z)^2 & x-y-z & x-y-z \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

$R_1 \to R_1 - (R_2 + R_3)$

$$\Delta = (x+y+z)^2\begin{vmatrix} 2yz & -2z & -2y \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

Determinant

Ex:- $\Delta = \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$

$$\Delta = \frac{xyz}{xyz}\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix}$$

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x(y+z)^2 & x^2y & zx^2 \\ xy^2 & (x+z)^2y & y^2z \\ xz^2 & yz^2 & (x+y)^2z \end{vmatrix}$$

$$\Delta = \frac{\cancel{xyz}}{\cancel{xyz}}\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$$

$\Rightarrow$ $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\Delta = \begin{vmatrix} (y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2 \\ y^2 & (x+z)^2-y^2 & 0 \\ z^2 & 0 & (x+y)^2-z^2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} (y+z)^2 & [x-(y+z)](x+y+z) & [x-(y+z)](x+y+z) \\ y^2 & (x+z-y)(x+z+y) & 0 \\ z^2 & 0 & (x+y-z)(x+y+z) \end{vmatrix}$$

$$\Delta = (x+y+z)^2\begin{vmatrix} (y+z)^2 & x-y-z & x-y-z \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

$R_1 \to R_1 - (R_2+R_3)$

$$\Delta = (x+y+z)^2\begin{vmatrix} 2yz & -2z & -2y \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

$\boxed{C_2 \to C_2 + \frac{1}{y}C_1,\ C_3 \to C_3 + \frac{1}{z}C_1}$ $\Rightarrow \Delta = (x+y+z)^2\begin{vmatrix} 2yz & 0 & 0 \\ y^2 & x+z & y^2/z \\ z^2 & z^2/y & x+y \end{vmatrix}$

$$\Delta = (x+y+z)^2\left[2yz\left\{(x+z)(x+y) - \frac{z^2}{y}\times\frac{y^2}{z}\right\}\right]$$

$$\Delta = (x+y+z)^2\left[2yz\left\{(x^2+xy+zx+\cancel{zy}) - \cancel{zy}\right\}\right]$$

$$\Delta = (x+y+z)^2\left[2xyz(x+y+z)\right] = (x+y+z)^3 \cdot 2xyz = RHS$$