Determinant

Ex:- if a, b, c are in A.P.

$$\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix} = 0$$

Sol^n:- $a, b, c \rightarrow AP$:-

$$\boxed{2b = a+c}$$

Now:- $R_2 \rightarrow 2R_2 - (R_1+R_3)$

$$\Rightarrow \begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 0 & 0 & (18y+2b)-(18y+a+c) \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 0 & 0 & 0 \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix} \rightarrow \text{here } R_2 \rightarrow 0$$

$$\Rightarrow \Delta = 0 \quad \text{Ans}$$

Ex:- $\Delta = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$ & $\Delta_1 = \begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$

✓A) $\Delta = 2\Delta_1$ B) $\Delta = -2\Delta_1$ C) $\Delta = 4\Delta_1$ D) $\Delta = -4\Delta_1$

Sol^n:- $R_1 \rightarrow R_1+R_2+R_3$

$$\Delta = \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$\Delta = 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} \quad R_2 \rightarrow R_2 - R_1,\ R_3 \rightarrow R_3 - R_1$$

$$\Delta = 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix} \Rightarrow R_1 \rightarrow R_1 + (R_2+R_3)$$

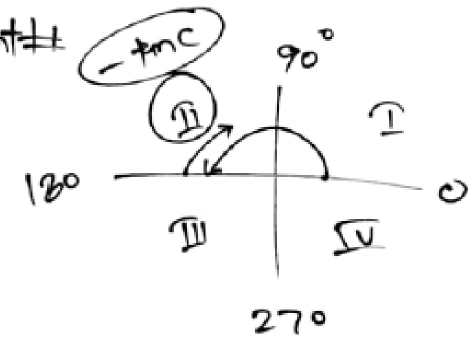
$$\Delta = 2\begin{vmatrix} a & p & x \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix} \Rightarrow +2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \Rightarrow C_3 \leftrightarrow C_1 \Rightarrow -2\begin{vmatrix} x & p & a \\ y & q & b \\ z & r & c \end{vmatrix}$$

$$\Rightarrow C_2 \leftrightarrow C_3 \Rightarrow \boxed{\Delta = +2\Delta_1} \quad \text{Ans}$$

Determinant

Ex:- if $\boxed{A+B+C=\pi}$ then:-

$$\begin{vmatrix} \tan(A+B+C) & \tan B & \tan C \\ \tan(A+C) & 0 & \tan A \\ \tan(A+B) & \tan A & 0 \end{vmatrix} = ?$$



A) 0 B) 1 C) $\tan A \tan B \tan C$ D) -2

Soln:- Solving through $R_1 \rightarrow$

$\Rightarrow \tan(A+B+C)(-\tan^2 A) - \tan B(\tan A)\cdot\tan(A+B) + \tan C\cdot\tan A\cdot\tan(A+C)$

$\Rightarrow \because \boxed{A+B+C=\pi}$

$\tan(A+B+C) = \tan\pi = 0$

$A+B = \pi - C$

$\tan(A+B) = \tan(\pi - C)$

$\tan(A+B) = -\tan C$

$\because A+C = \pi - B$

$\tan(A+C) = \tan(\pi - B)$

$\tan(A+C) = -\tan B$

So:- $\Rightarrow 0 - \tan B\cdot\tan A\cdot(-\tan C) + \tan C\cdot\tan A\cdot(-\tan B)$

$= \tan A \tan B \tan C - \tan A \tan B \tan C = 0$

Determinant

Ex: $A = \begin{vmatrix} 1 & \log_b a \\ \log_a b & 1 \end{vmatrix} = ?$

A) ab B) 0 C) ba D) 1

Sol^n:- $\Rightarrow 1 - \log_b a \cdot \log_a b = ?$

$\because \log_a b \rightarrow \dfrac{1}{\log_b a}$

$\Rightarrow 1 - \cancel{\log_b a} \cdot \dfrac{1}{\cancel{\log_b a}} = 1 - 1 = 0$ Ans

Ex:- if $\begin{vmatrix} a & 2b & 2c \\ 3 & b & c \\ 4 & a & b \end{vmatrix} = 0$ then find $abc = ?$

A) $a+b+c$ B) 6 C) b^3 D) $ab+b-c$

Sol^n:- $a(b^2 - ac) - 2b(3b - 4c) + 2c(3a - 4b) = 0$

$\Rightarrow ab^2 - a^2c - 6b^2 + \cancel{8bc} + 6ac - \cancel{8bc} = 0$

$\Rightarrow \underset{1}{ab^2} - \underset{2}{a^2c} - \underset{3}{6b^2} + \underset{4}{6ac} = 0$

$\rightarrow b^2(a-6) + 6ac - a^2c = 0$

$\rightarrow b^2(a-6) + ac(6-a) = 0$

$\Rightarrow b^2(a-6) - ac(a-6) = 0$

$\Rightarrow (a-6)[b^2 - ac] = 0$

$\Rightarrow b^2 - ac = 0$

$\Rightarrow \boxed{b^2 = ac}$

Multiply $b \rightarrow$ $\Rightarrow \boxed{b^3 = abc}$ Ans

Determinant

Ex. if ω is cube root of unity

then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = ?$

Solⁿ :- $\because$ $\omega \to$ cube root of unity.

$\sqrt[3]{\omega} = 1 \Rightarrow \boxed{\omega^3 = 1}$

$\Rightarrow \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} \Rightarrow C_1 + C_2 + C_3 \to C_1$

$\Rightarrow \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$

$(1+\omega+\omega^2)\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & 1 \\ 1 & 1 & \omega \end{vmatrix}$

$\Rightarrow (1+\omega+\omega^2)\left[1(\omega^3-1) - \omega(\omega-1) + \omega^2(1-\omega^2)\right]$

$\Rightarrow (1+\omega+\omega^2)\left[\omega^3 - 1 - \cancel{\omega^2} + \omega + \cancel{\omega^2} - \omega^4\right]$

$\Rightarrow (1+\omega+\omega^2)\left[\omega^3 - 1 + \omega(1-\omega^3)\right]$

$\Rightarrow (1+\omega+\omega^2)\left[\cancel{1} - \cancel{1} + \omega(\cancel{1} - \cancel{1})\right]$

$\Rightarrow (1+\omega+\omega^2)\left[0\right] = 0$ Ans